Vehicle Rotation Planning for ICE High Speed Trains

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joint work with O. Heismann, M. Reuther, T. Schlechte, S. Weider et. al.

RobustRailS Mini Conference 2015

DTU, Copenhagen, 27.08.2015

InterCity Express (ICE) High Speed Train

… must be used efficiently

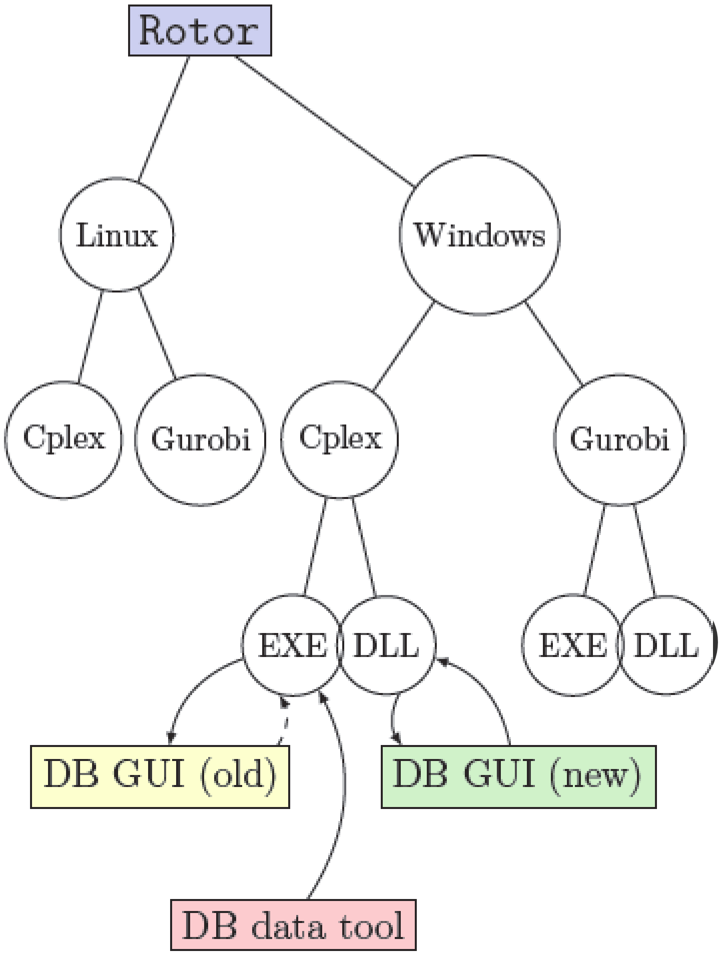
ROTOR Optimization Kernel

Rotor 1.0

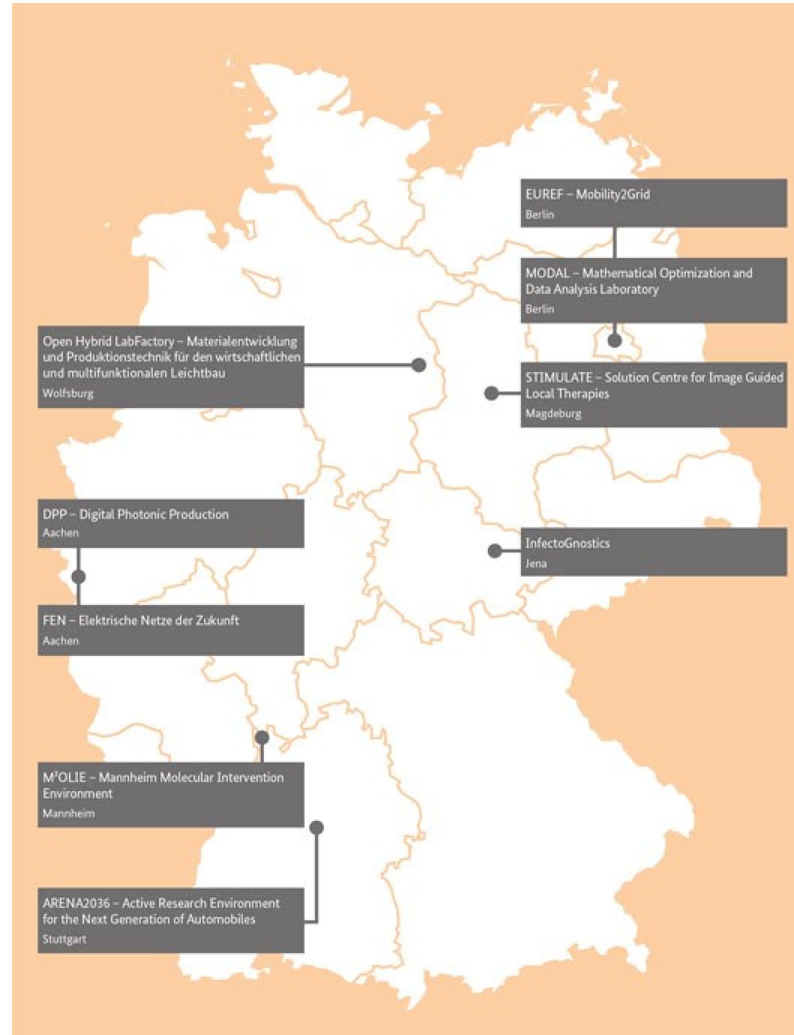
- in production since 7 / 2013
- integrates all technical details

Rotor 2.0

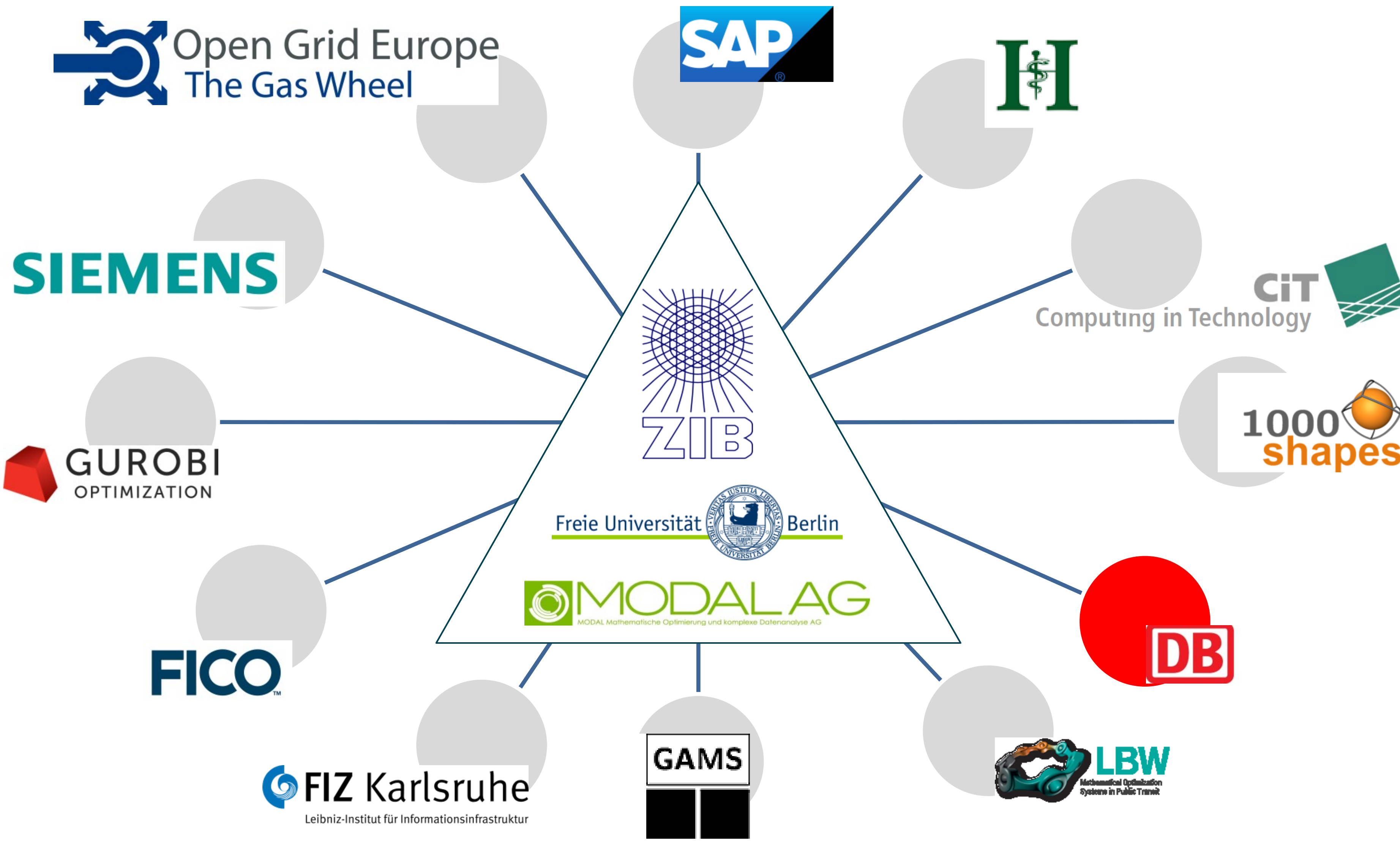
- in production since 3 / 2014
- reduced memory consumption
- implements re-optimization



Research Campus MODAL @ ZIB



MODAL: Industry Partners



MODAL:
Mathematical Optimization & Data Analysis Laboratories

RailLab
Fully dated, service oriented, vehicle rotation planning and dispatching

MedLab
Information based medical diagnostics

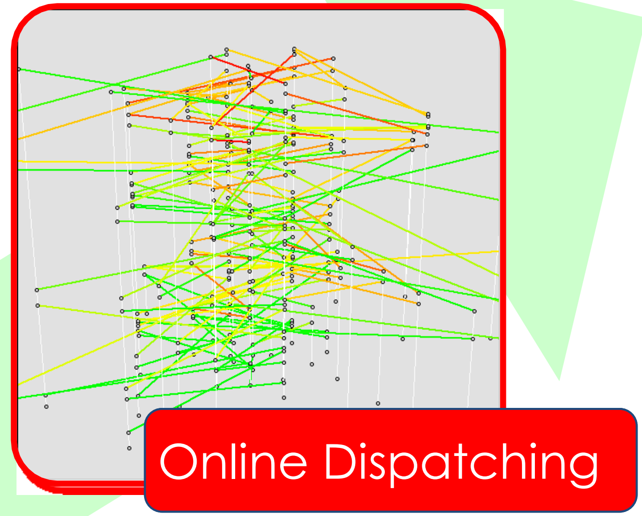
MODAL

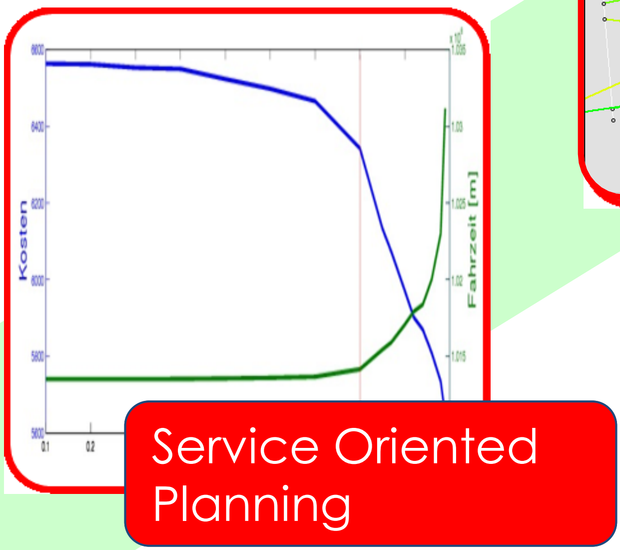
GasLab
Dynamic nomination validation

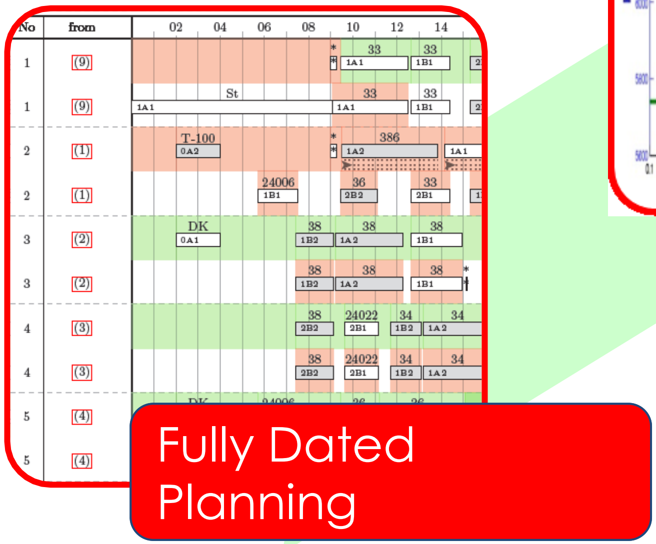
SynLab
Basic methods for discrete-continuous-stochastic optimization problems

MODAL Rail-Lab: Phases

- Builds on earlier work with DB (VR-OPT)
- 3 phases of 5 years, phase I 2015-2019
- 4 positions, 2 industry + 2 BMBF







Rail Lab Phase I: Topics

Fully Dated Vehicle rotation planning

Mathematical Models and Algorithms

Integer Programming
Integrated Flow and Path Model
Bundle Method
Coarse-to-Fine Method

Combinatorial Optimization
Hyperassignments
Hyperflows
Tree Decompositions

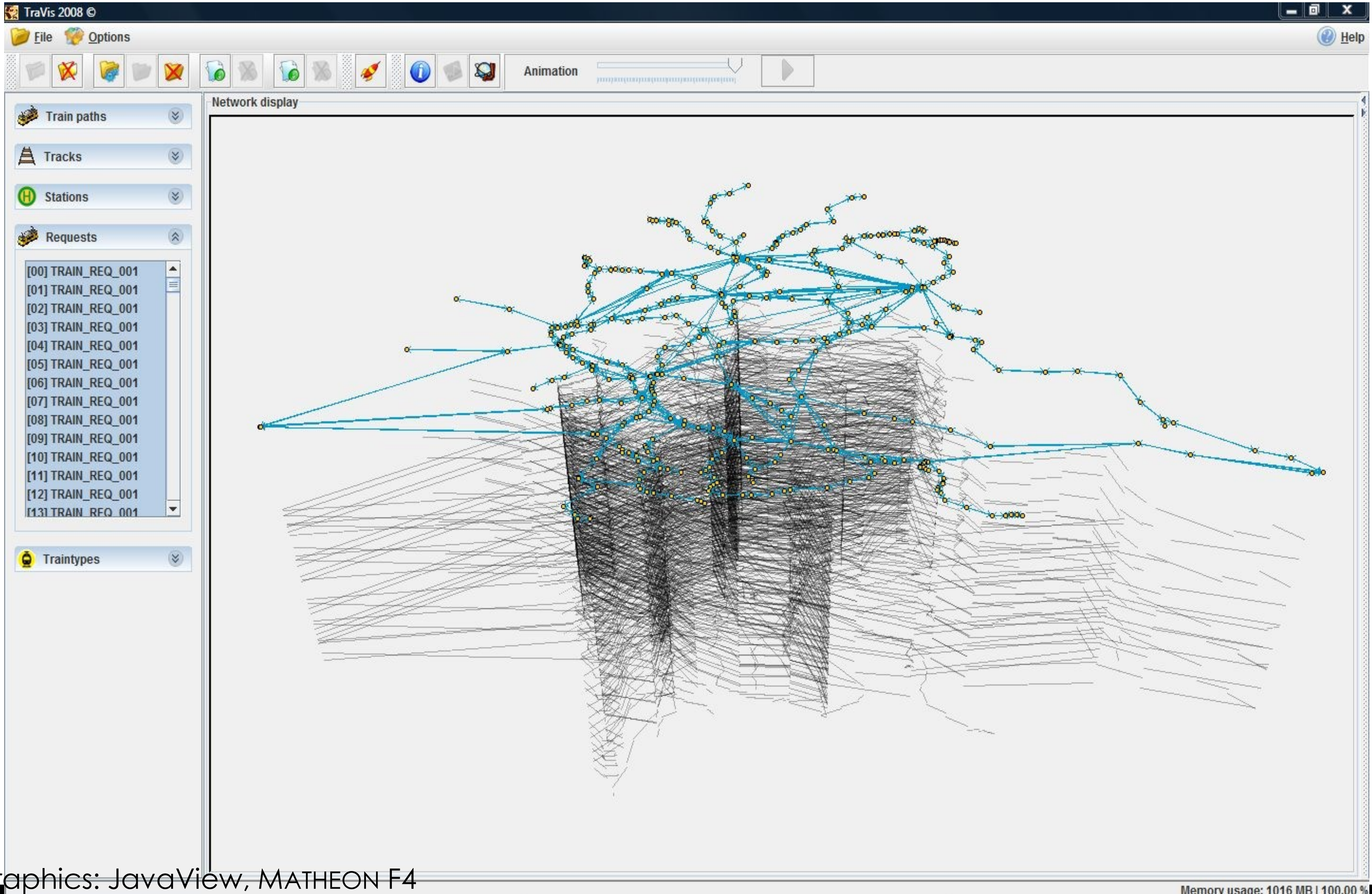
Data Analysis and System Integration

System Integration
Data acquisition
Interfaces
Optimization cores

Data Analysis
Railway Requirements
Case studies and calibration
Visualization and statistics

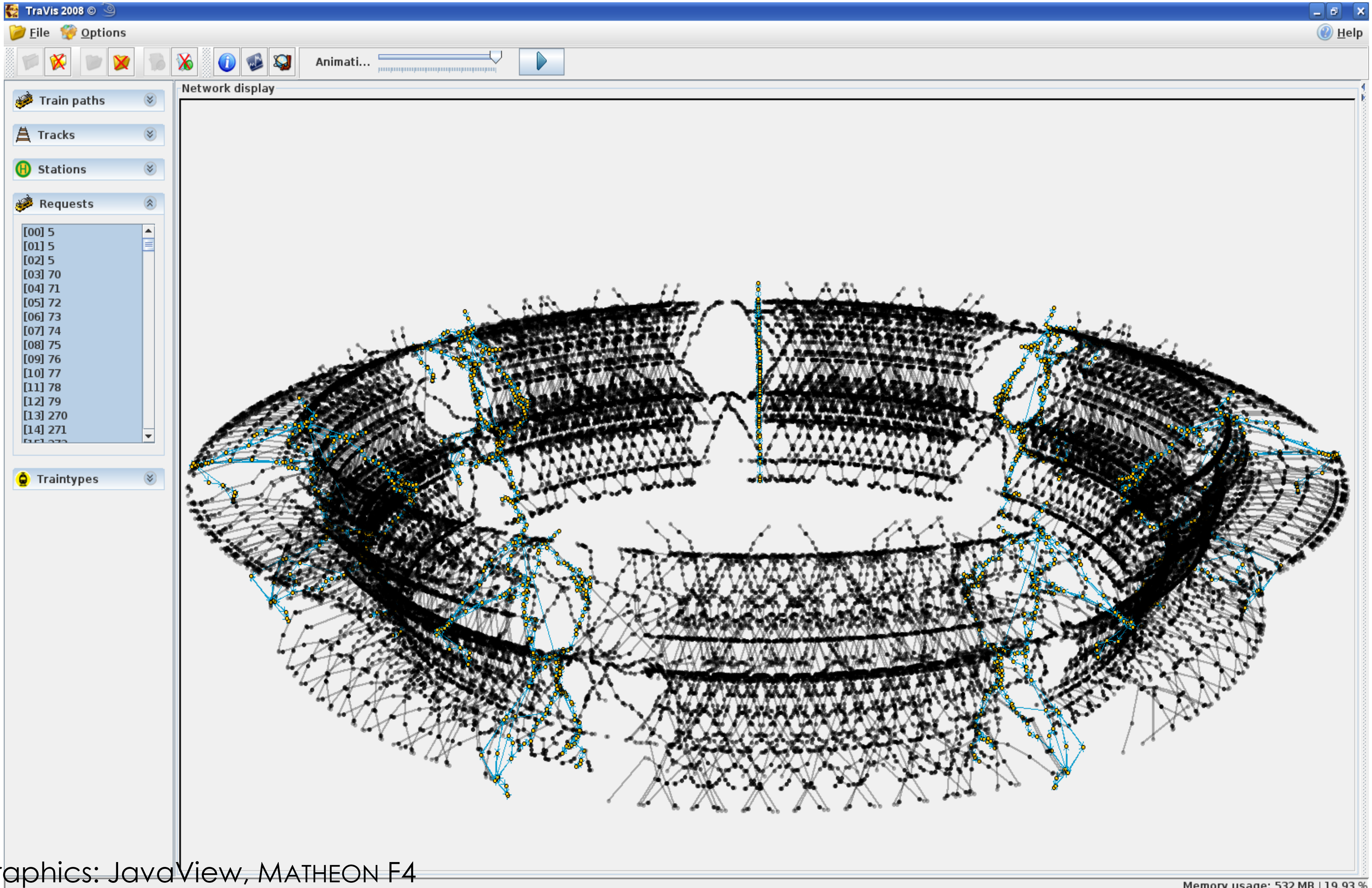
ICE Network: Connections

Timetabled Trips: 1 Day



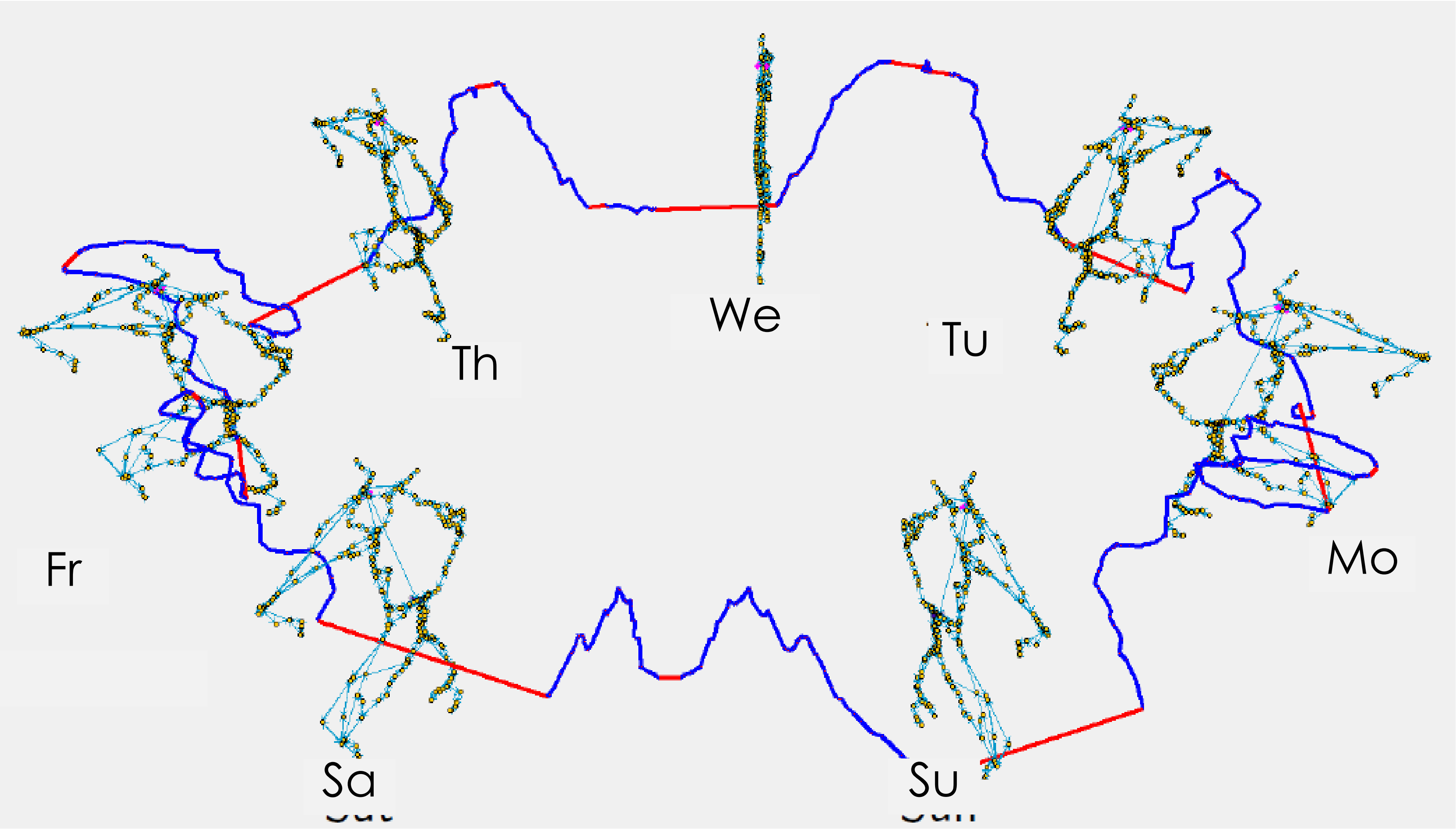
Graphics: JavaView, MATHEON F4

Timetabled Trips: Standard Week



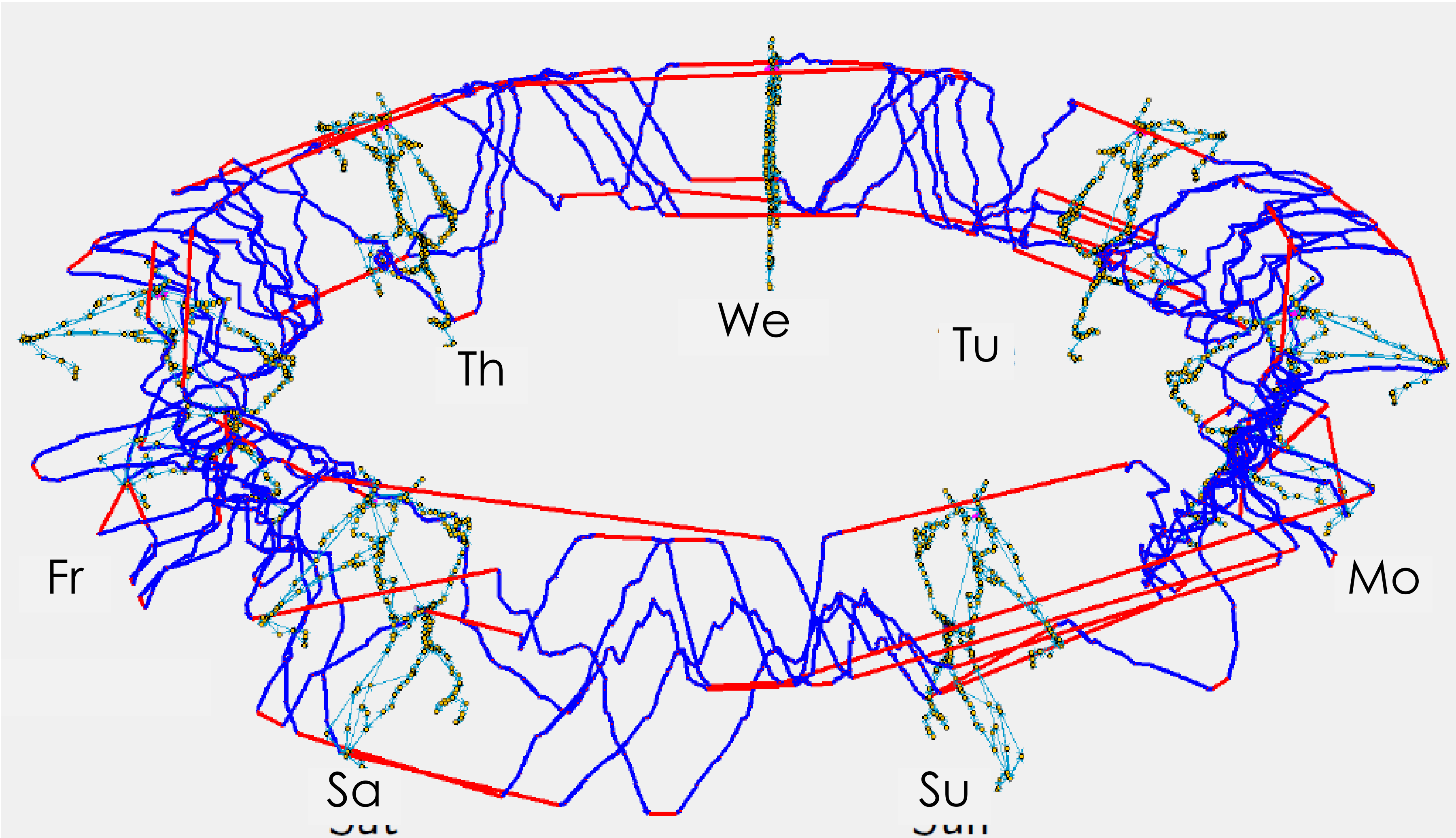
Graphics: JavaView, MATHEON F4

Vehicle Rotation: 1 Week



Graphics: JavaView, MATHEON F4

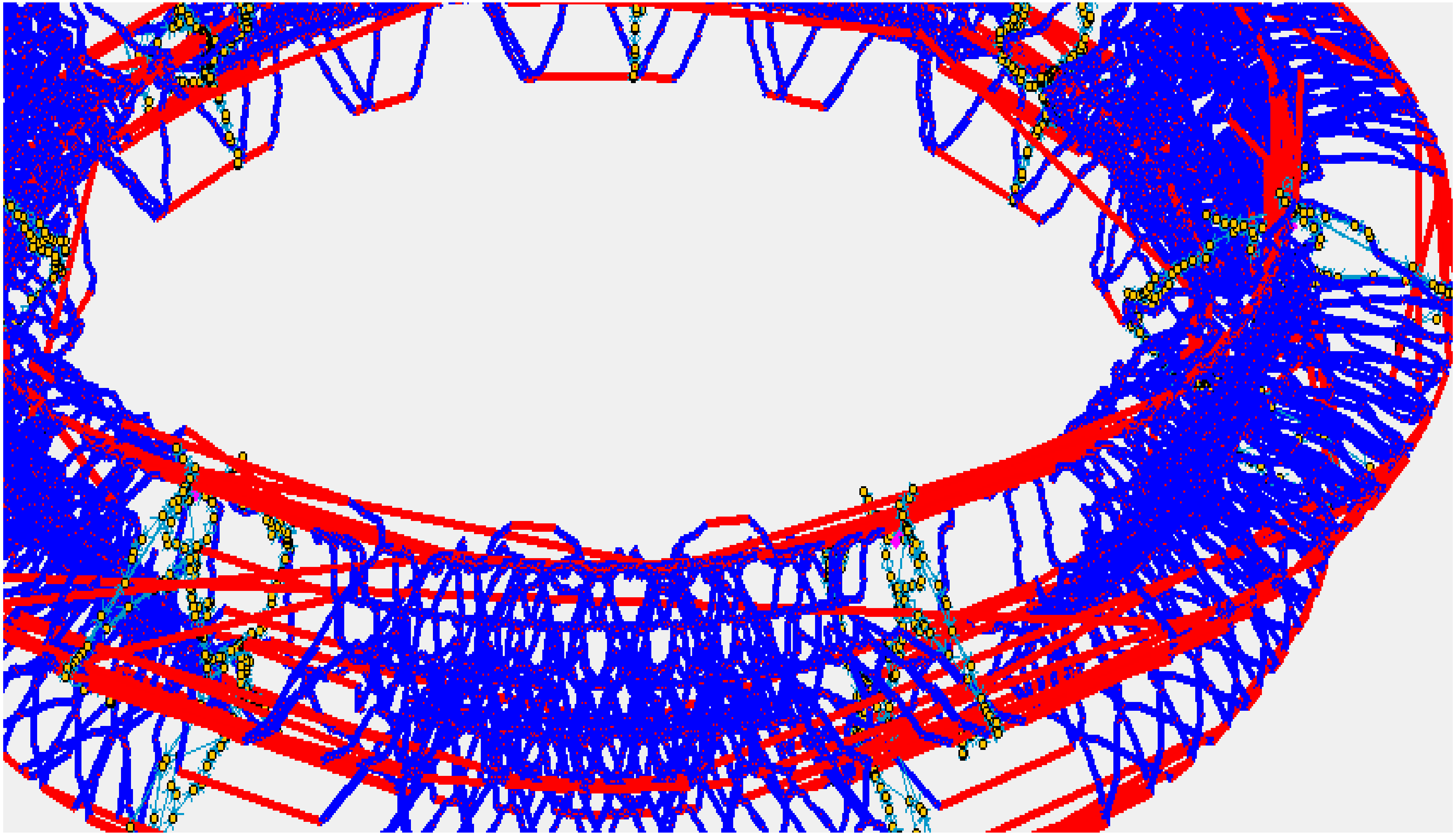
Vehicle Rotation: 5 Weeks



Graphics: JavaView, MATHEON F4

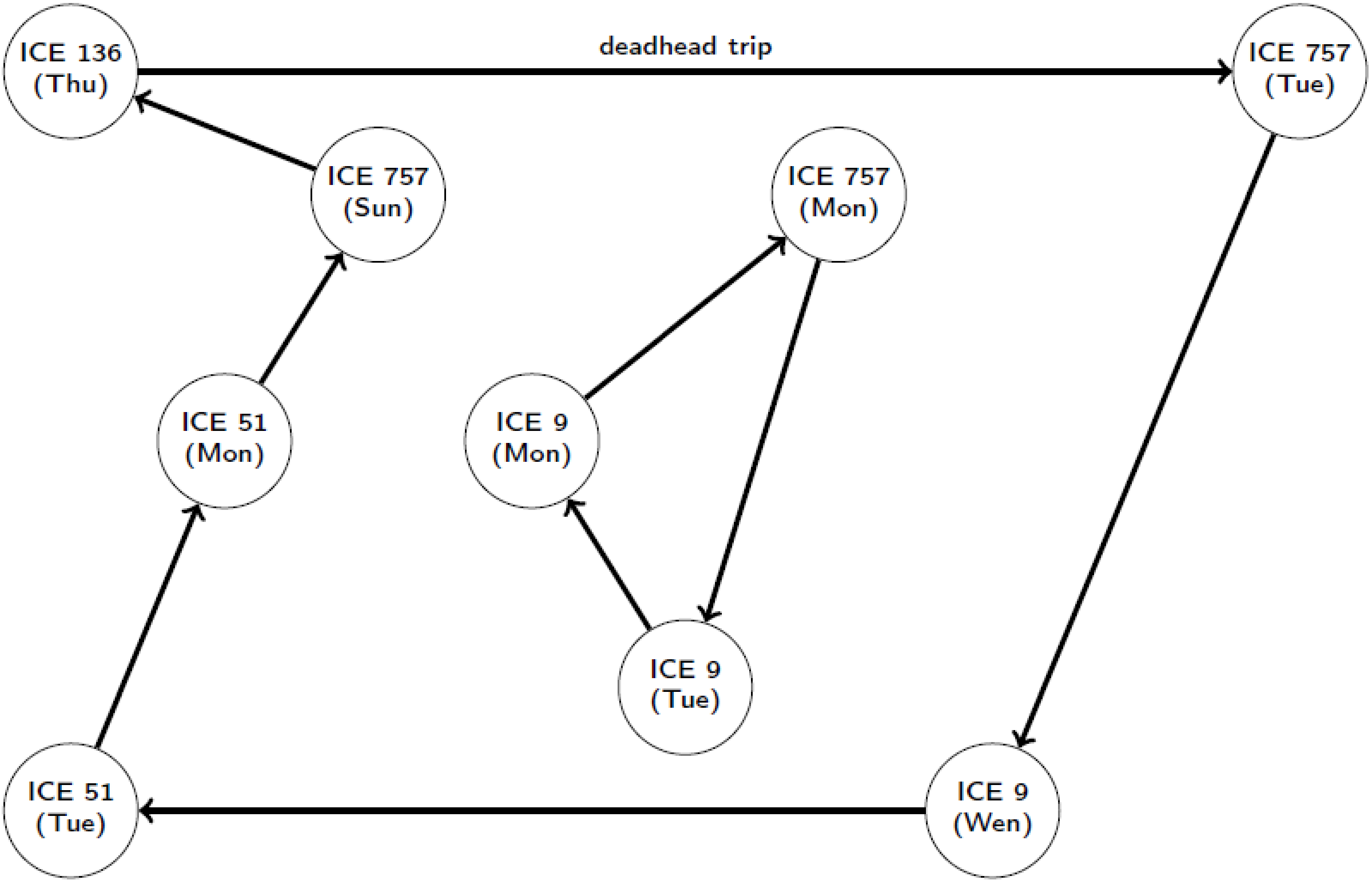
Rotation Plan: Follow-on Trip Assignment

(Blue: Timetabled Trips, Red: Deadhead Trips)

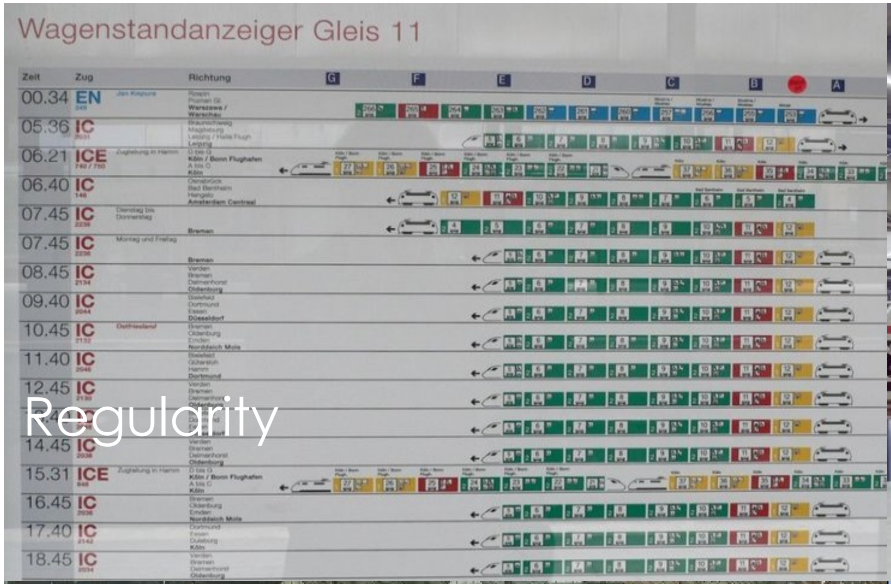


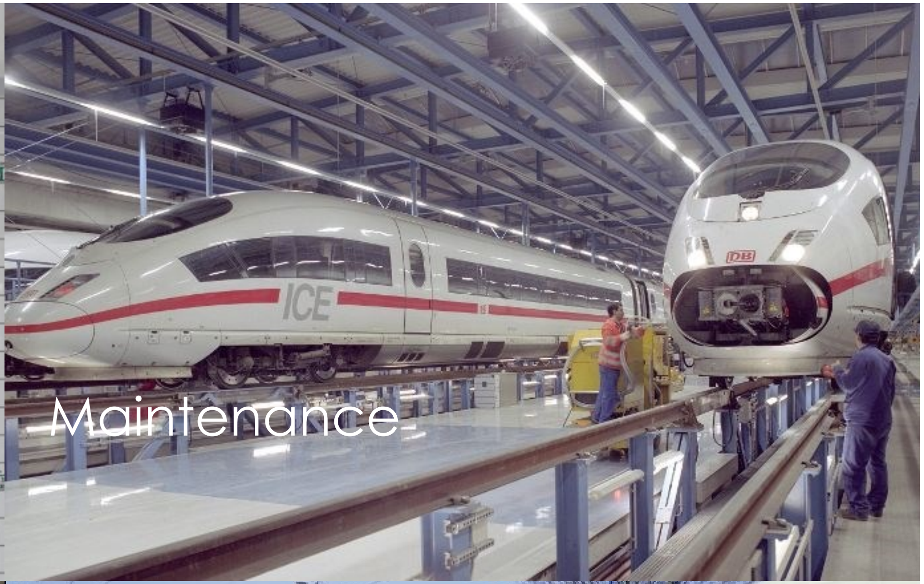
Graphics: JavaView, MATHEON F4

Again: Follow-on Trip Assignment



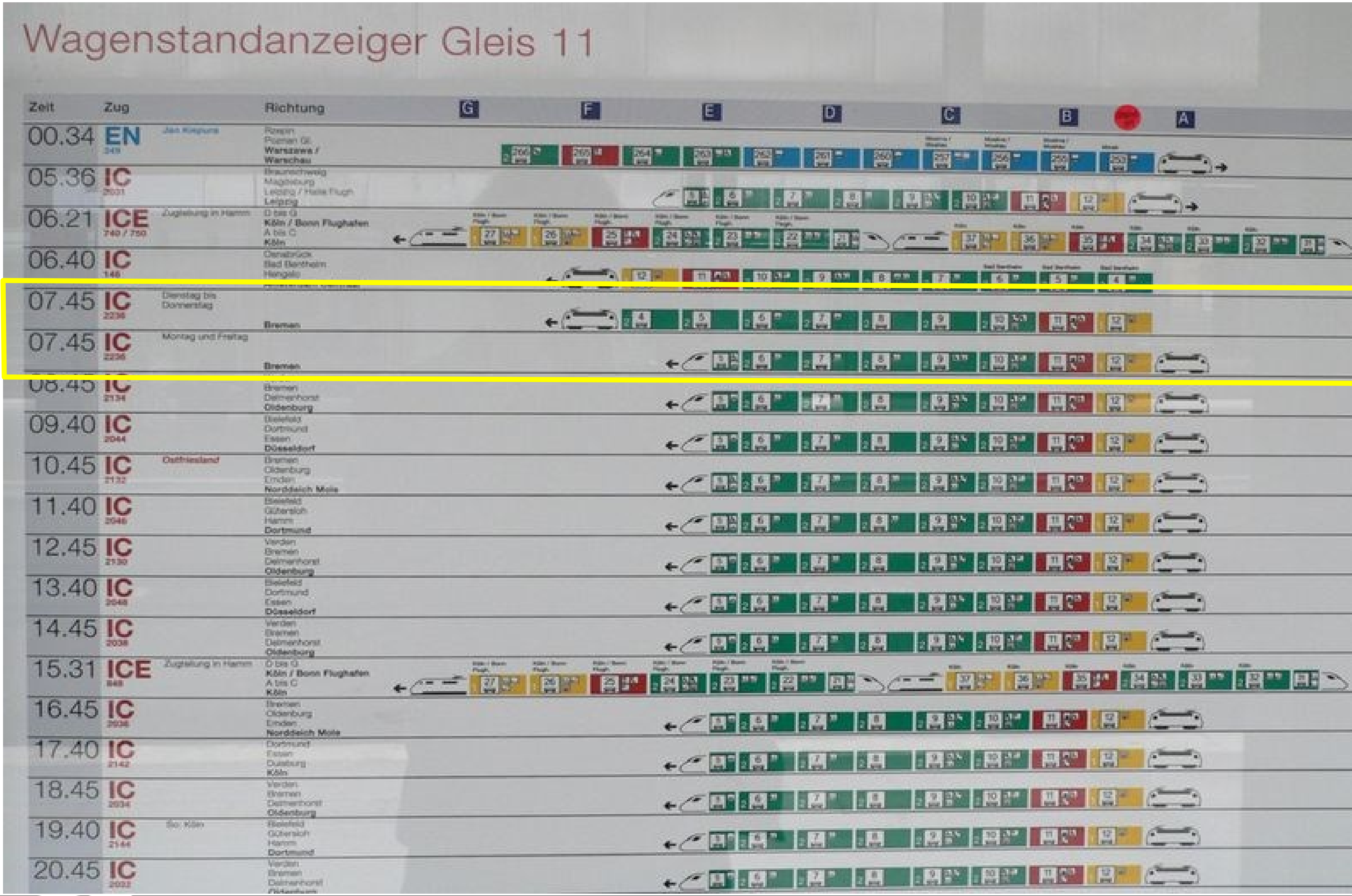
Railway Constraints





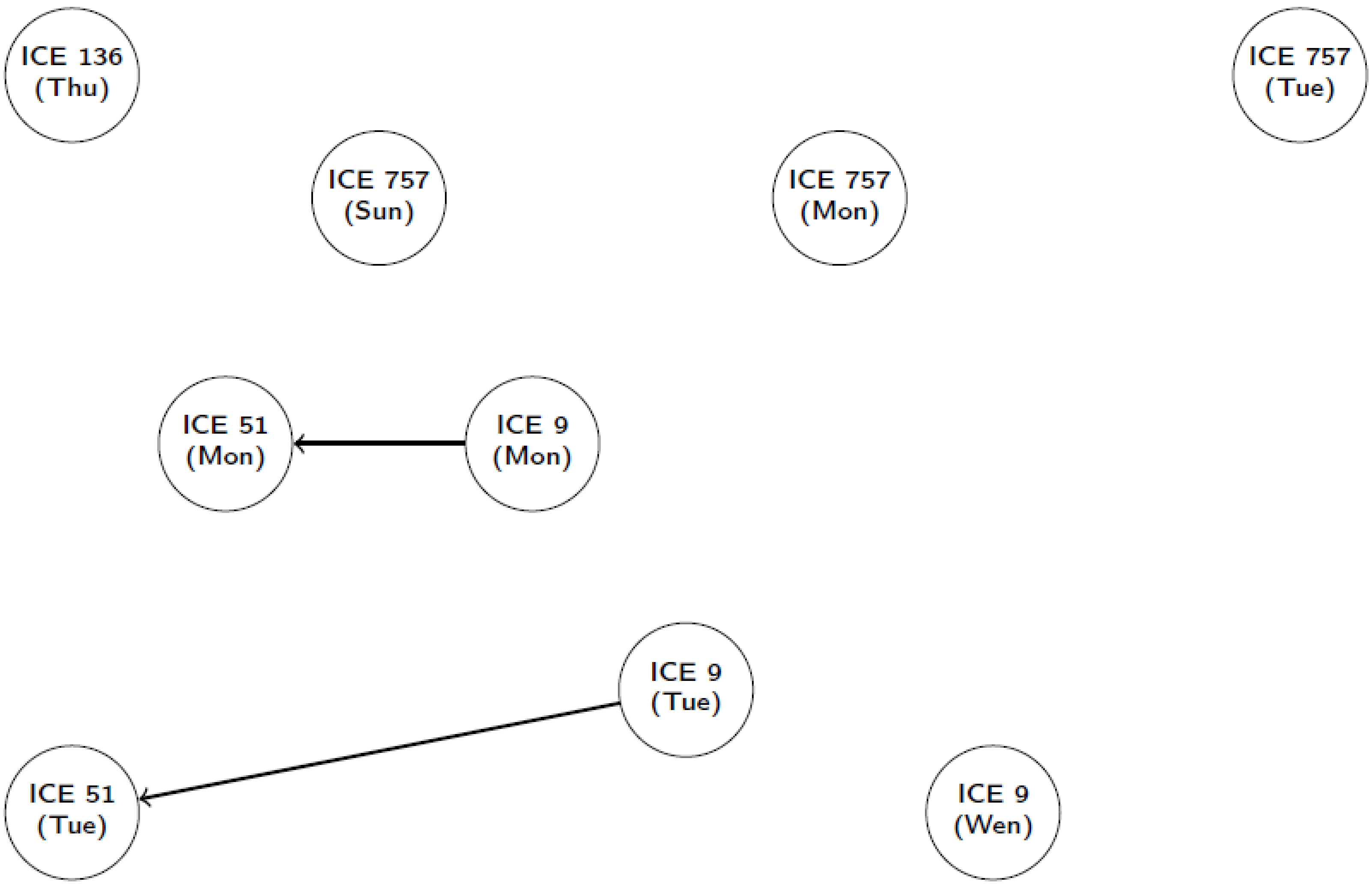
Photos courtesy of DB Mobility Logistics AG

Timetable Regularity → Rotation Regularity

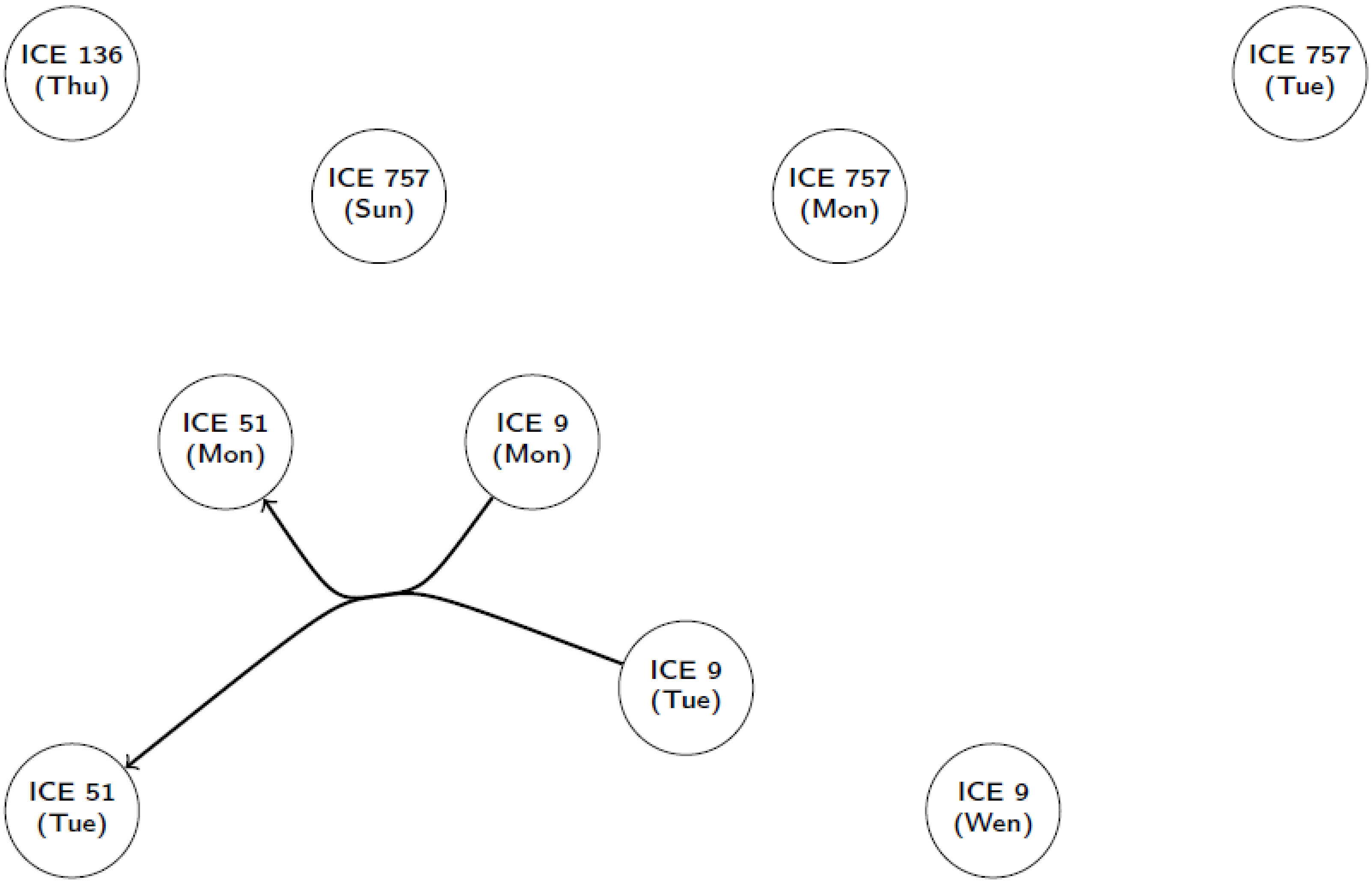


MODAL

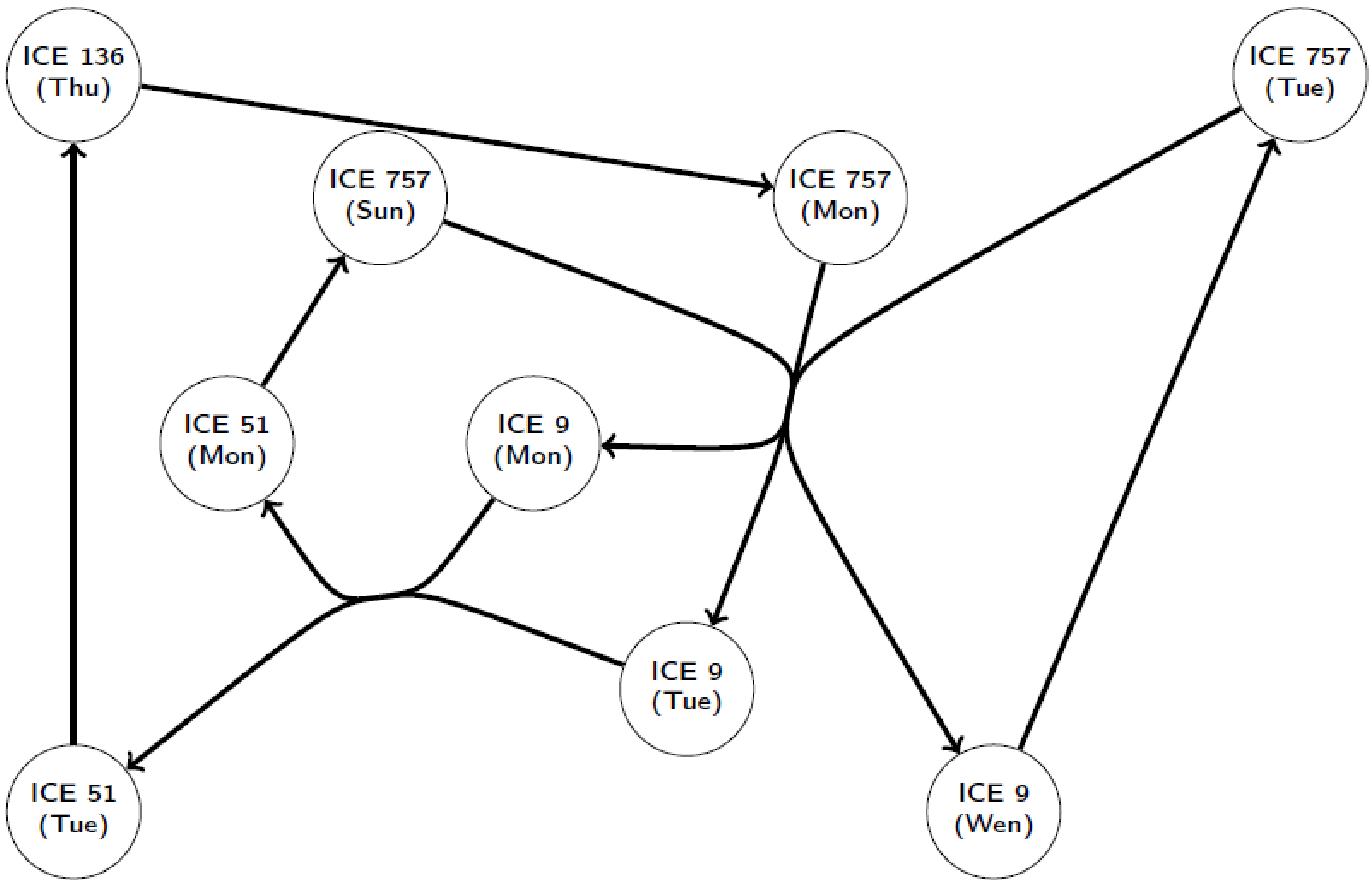
Modeling Rotation Regularity ...



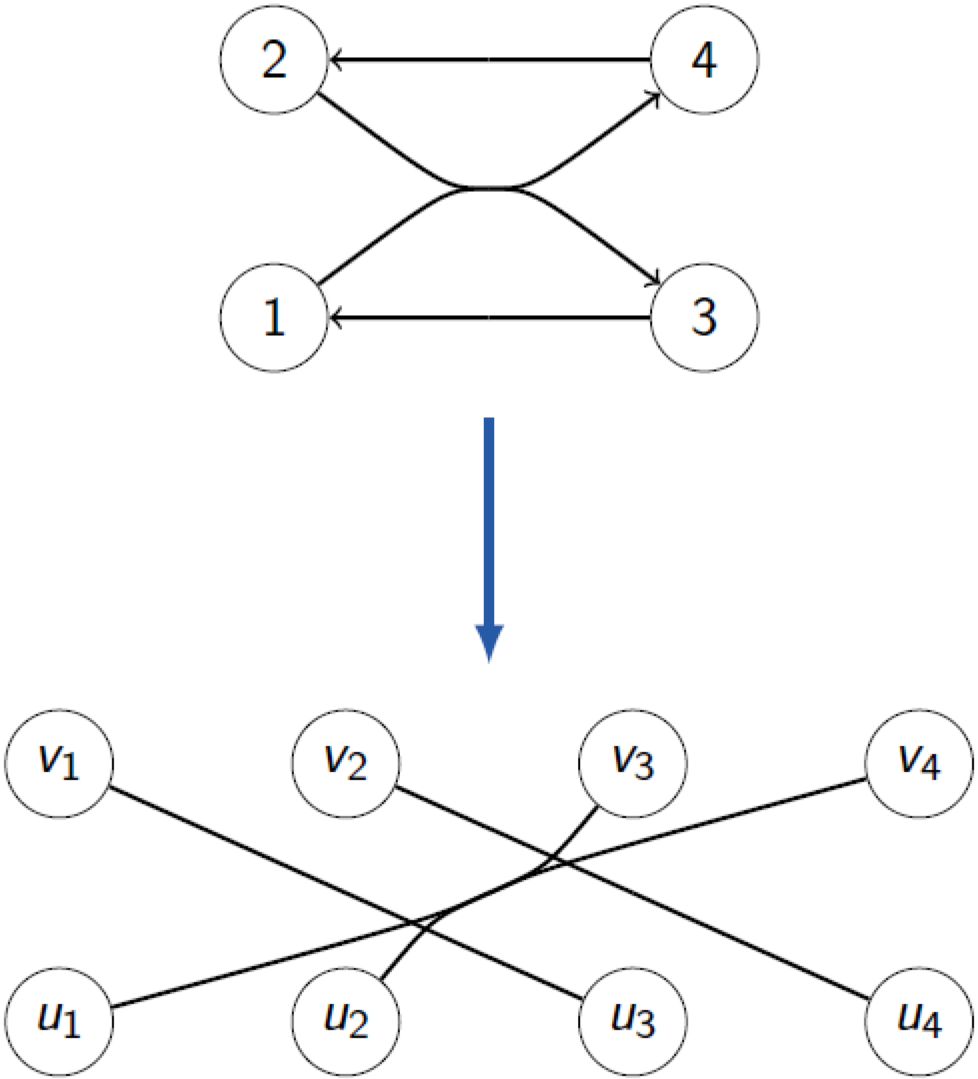
Modeling Rotation Regularity via Hyperarcs



Hyperassignment Solution



Bipartite Hypergraph Model

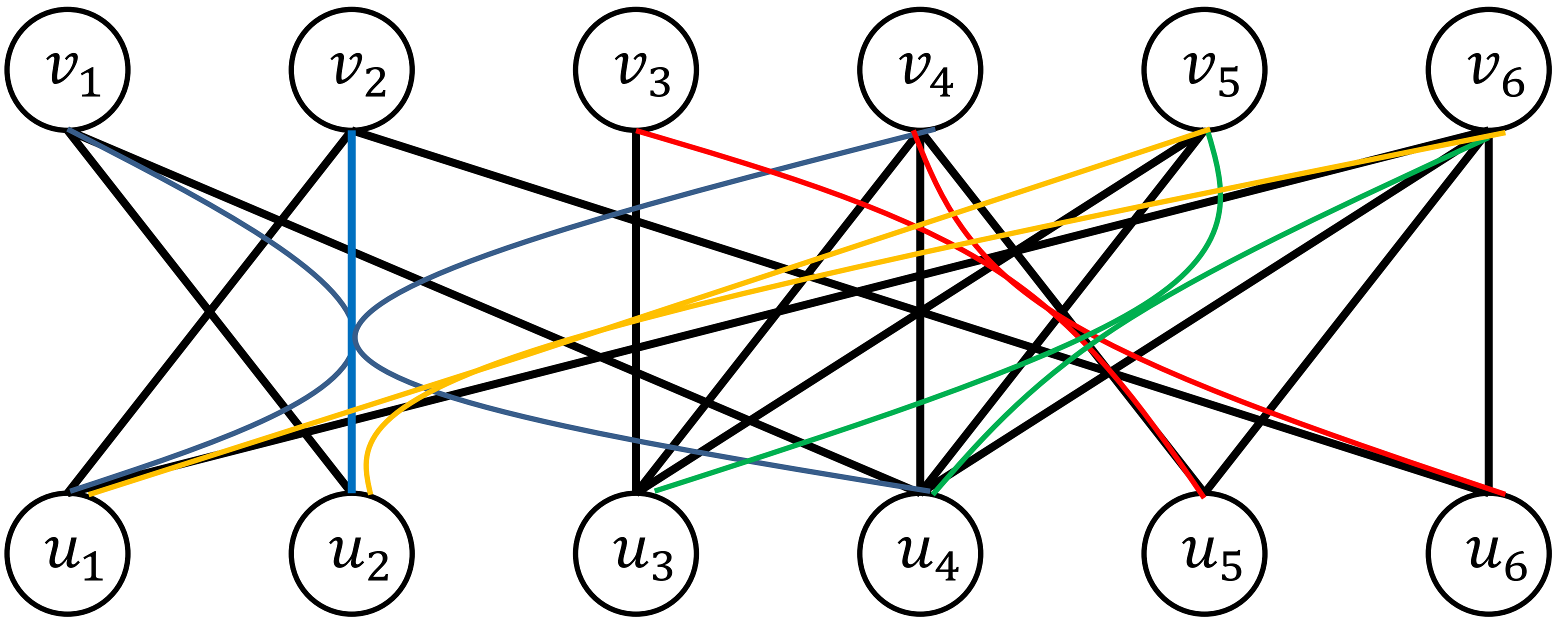


Bipartite Hypergraphs

A hypergraph G is called bipartite if

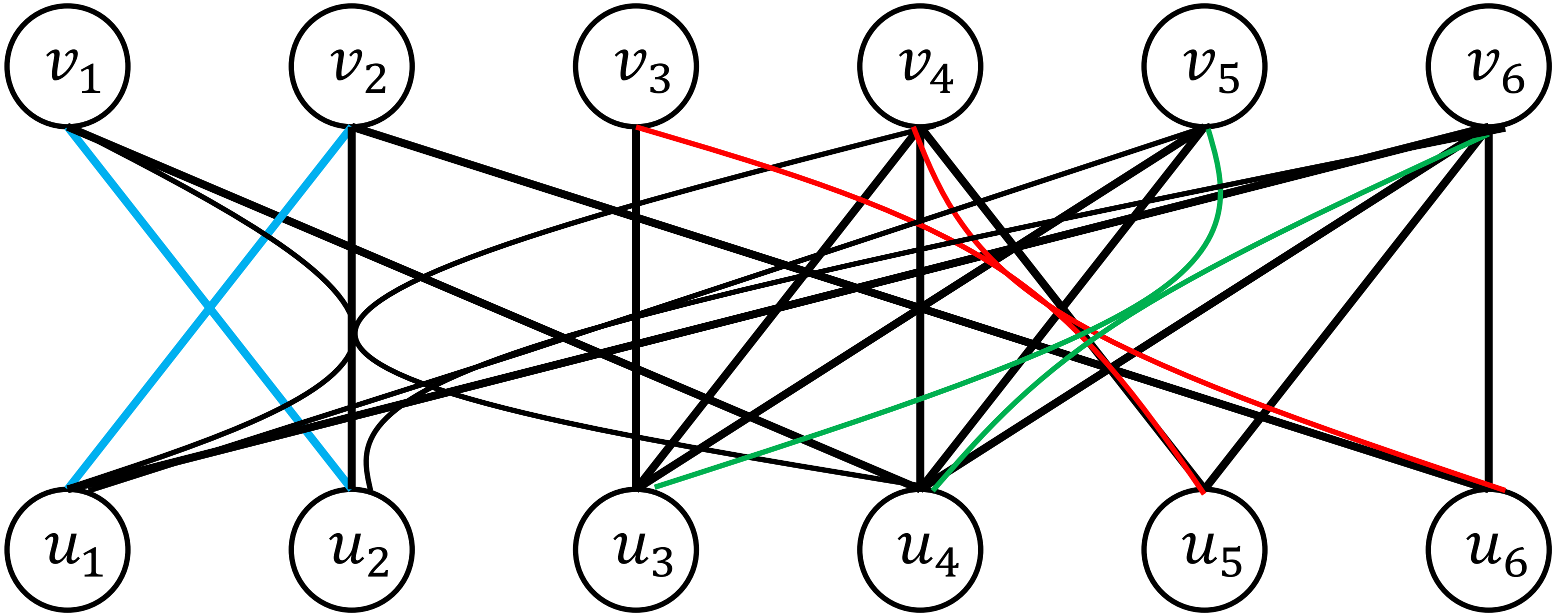
- its vertex set can be written as the disjoint union of two vertex sets U and V with the same size $|U| = |V|$, and
- every hyperedge $e \in E$ has the same number $|e \cap U| = |e \cap V|$ of vertices in U and V.

We then represent G as a triple $G = (U, V, E)$.



Hyperassignments

A hyperassignment is a subset H of E such that there is exactly one incident hyperedge for every vertex.



The Hyperassignment Problem

Definition (Hyperassignment Problem)

Input: A bipartite hypergraph $G = (U, V, E)$ with edge costs $c_e \in \mathbb{R}$.
Output: A minimum cost hyperassignment H^* in G, i.e., a hyper-assignment H^* s.t.

$$c(H^*) = \min\{c(H), H \text{ is a hyperassignment in } G\}$$

or the statement that no hyperassignment exists.

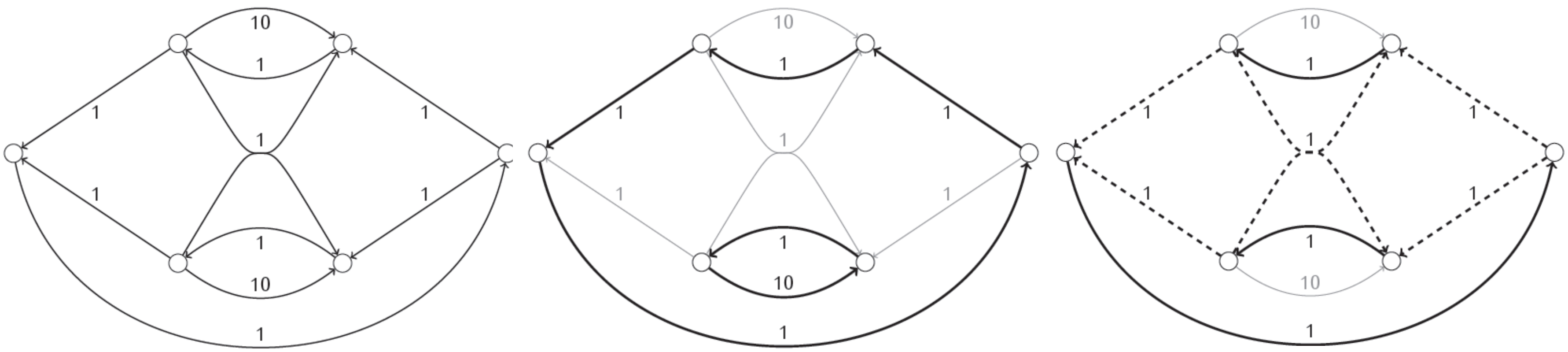
$$\begin{aligned} \min \quad & c^T x & & \\ & x(\delta^+(v)) = & 1 & \quad \forall v \in U \cup V \\ & x(\delta^-(v)) = & 1 & \quad \forall v \in U \cup V \\ & x & \in \{0,1\}^E & \end{aligned}$$

The HAP is a special type of set partitioning problem.

Complexity Results

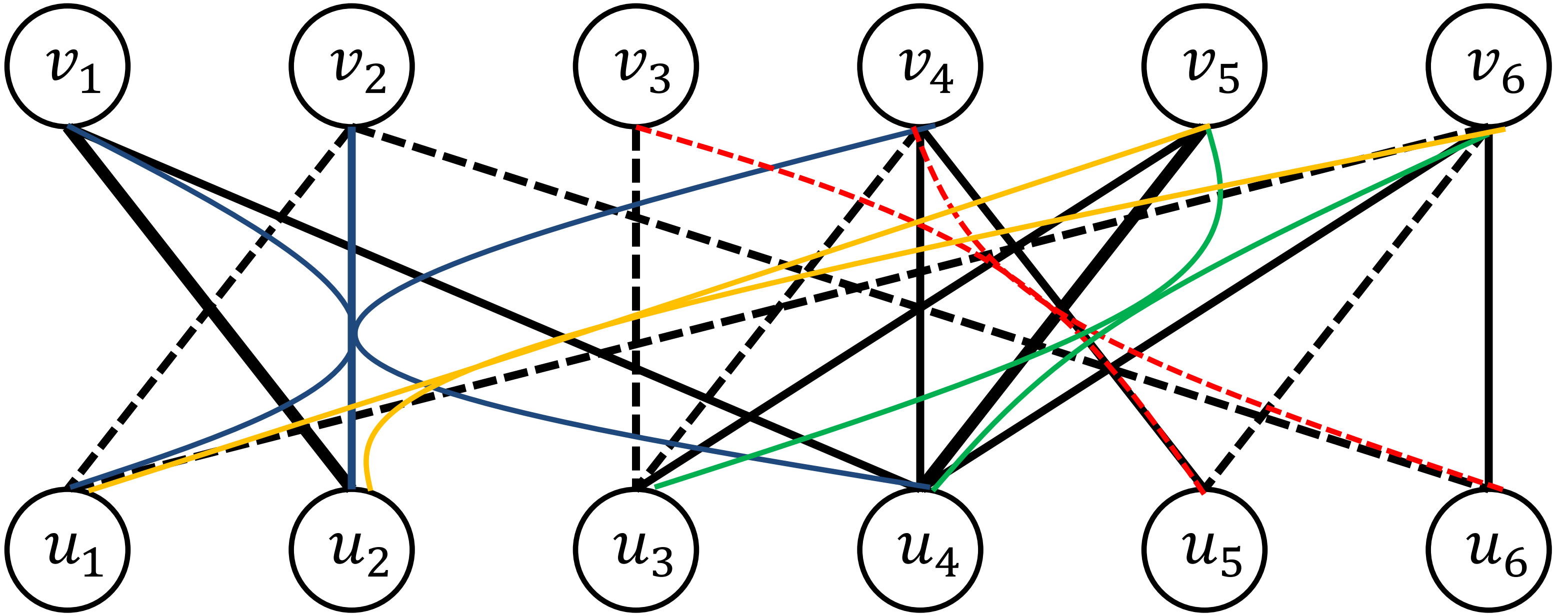
Theorem (B., Heismann [2011], Heismann [2014])

1. The HAP is NP-hard and APX-hard, even for bipartite hypergraphs with maximum hyperedge size 4.
2. The set packing/covering relaxations of the HAP are NP-hard, even for bipartite hypergraphs with maximum hyperedge size 6.
3. The LP/IP gap can be arbitrarily large.
4. The determinants of basis matrices can be arbitrarily large.



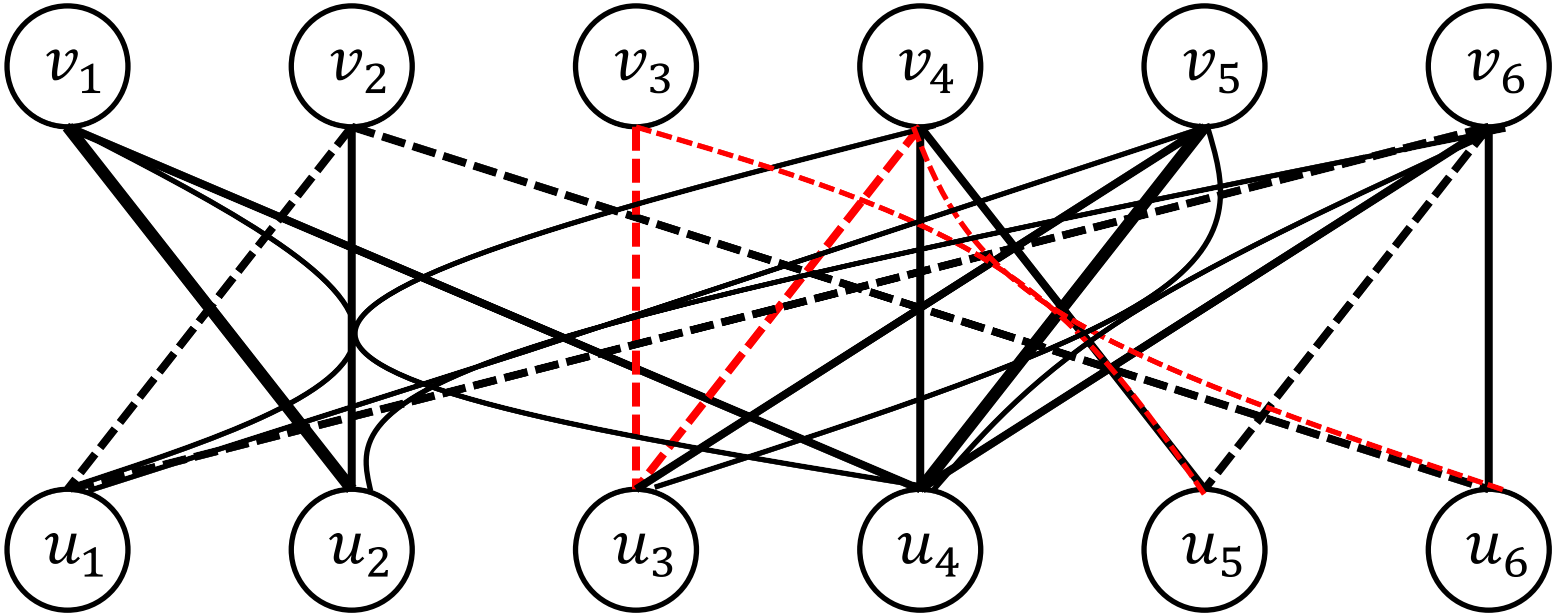
Solution of the LP Relaxation

- Fractional solution, cost = 0.615.

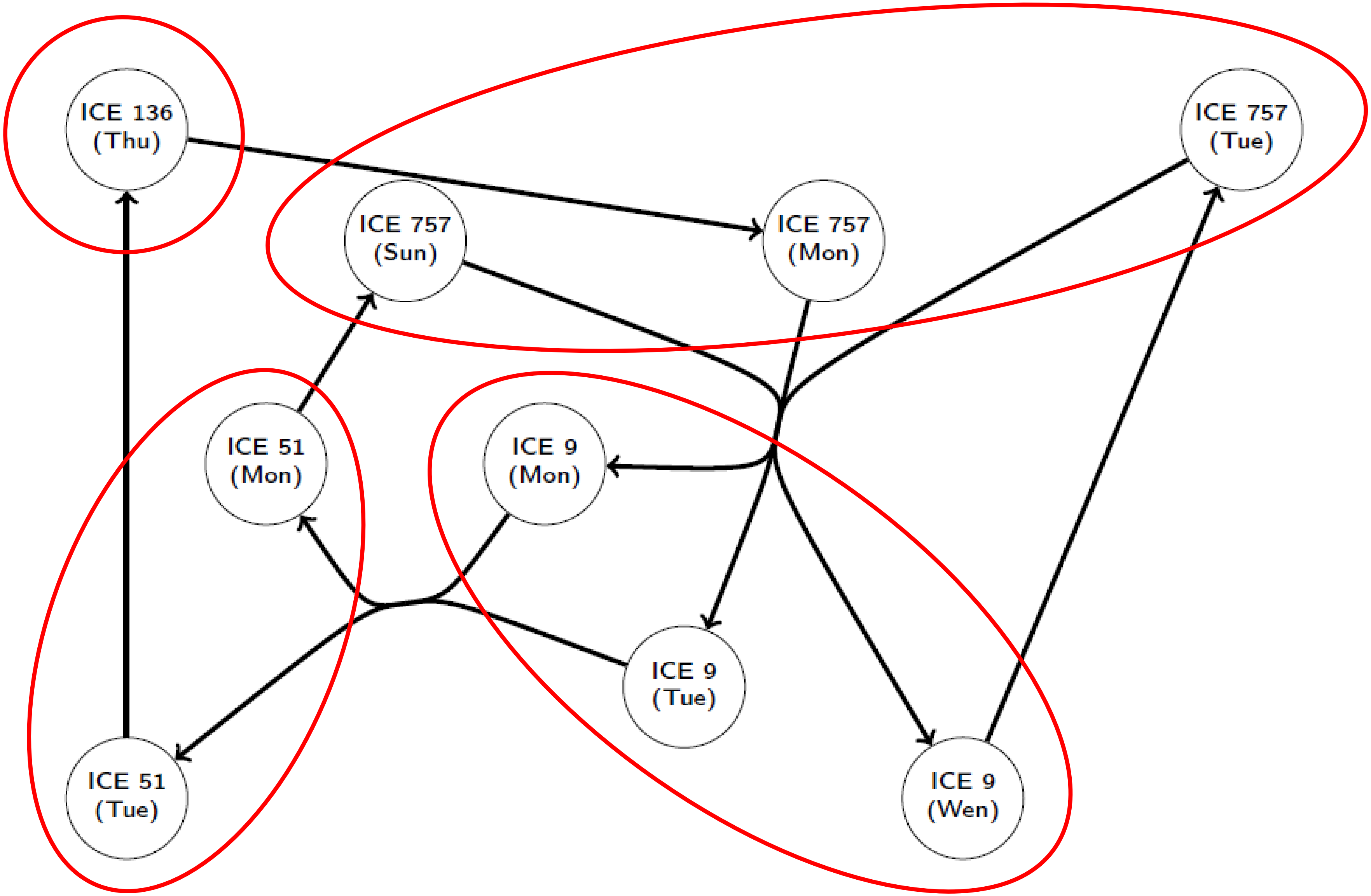


Solution of the LP Relaxation

- Fractional solution, cost = 0.615.
- The red hyperedge clique inequality separates this solution.
- Cliques can be separated efficiently by exploiting a "partitioning structure".



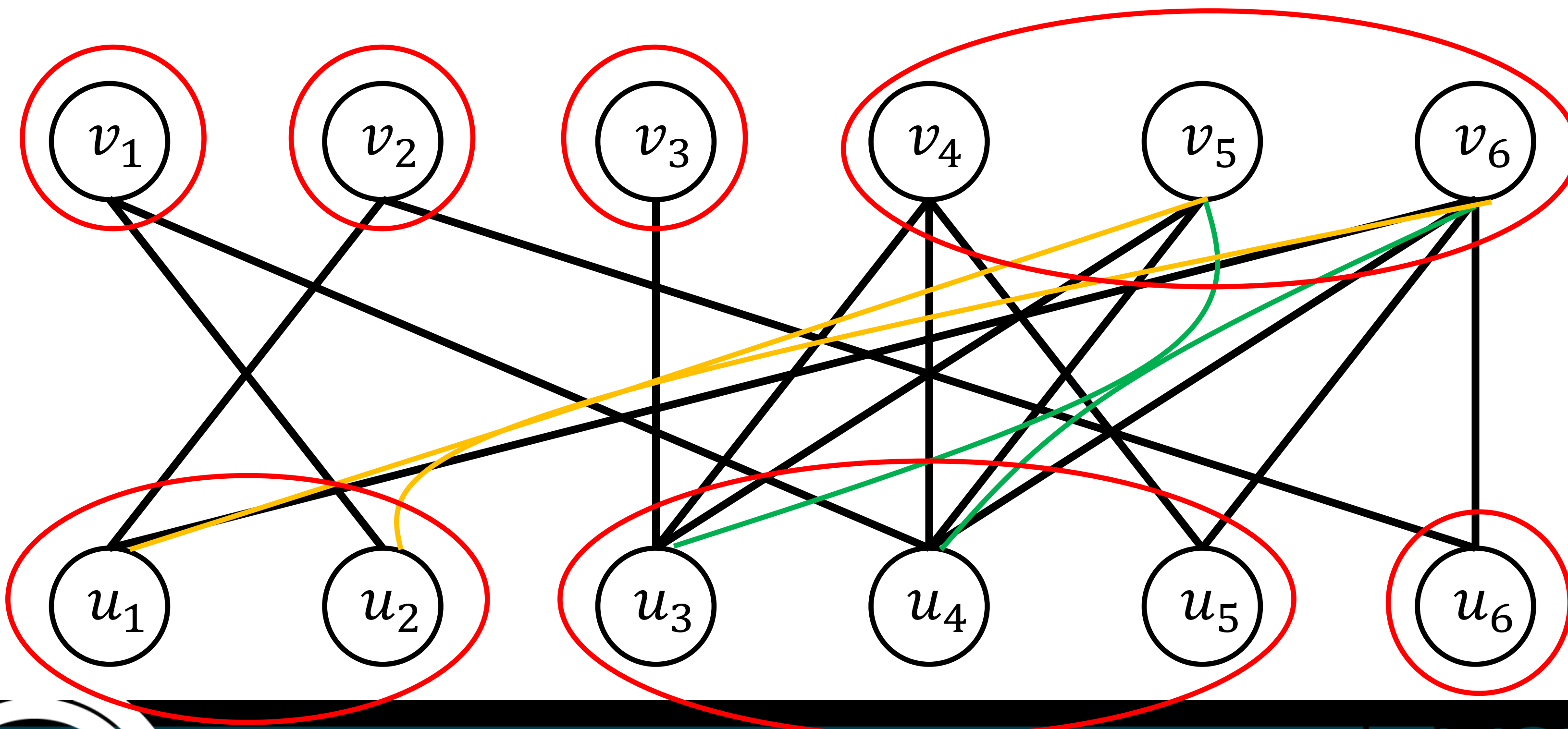
Partitioned Hypergraphs



Partitioned Hypergraphs

Theorem (B., Heismann [2012])

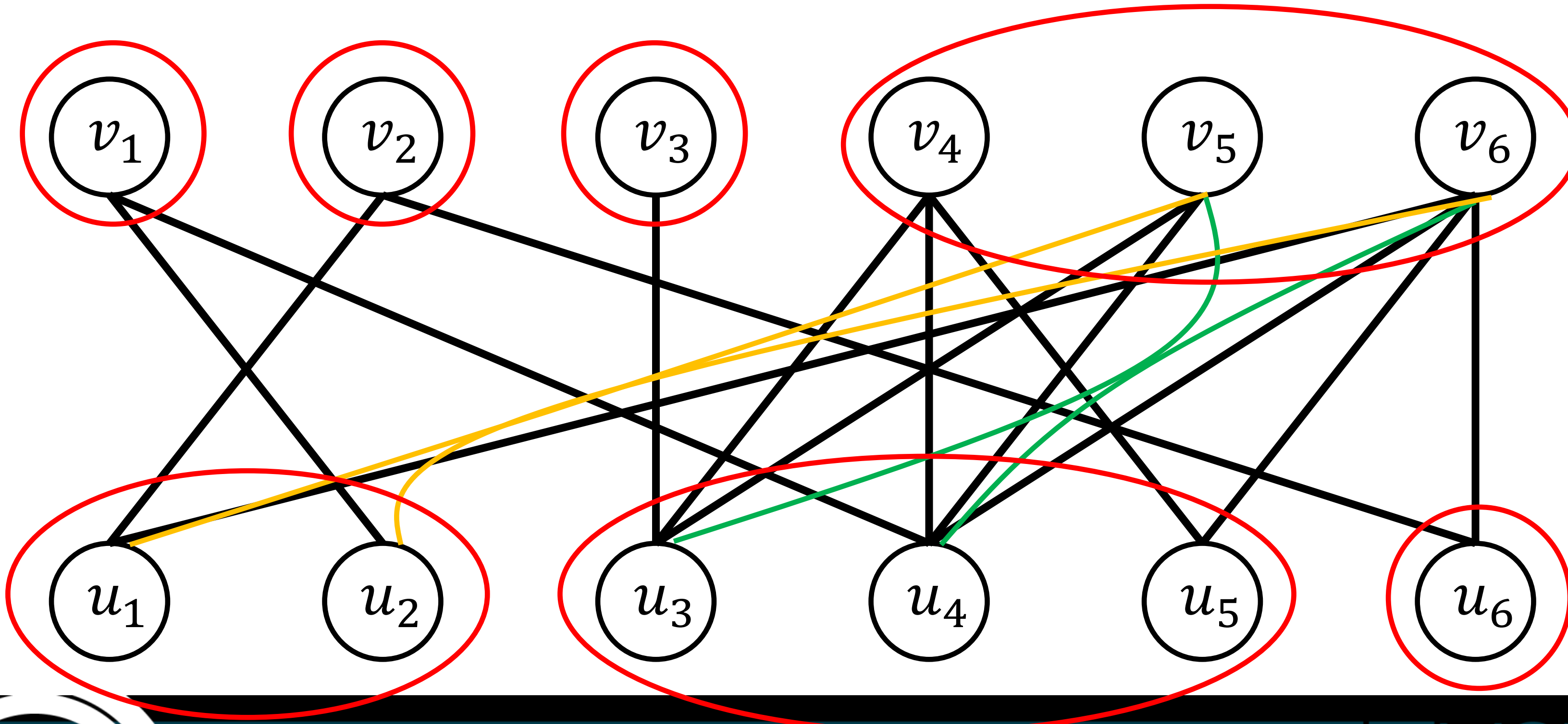
Every HAP in a bipartite hypergraph $G = (U, V, E)$ can be polynomially transformed into a HAP in a partitioned hypergraph with $d = 0.5 \max_{e \in E} |e|$.



Partitioned Hypergraphs

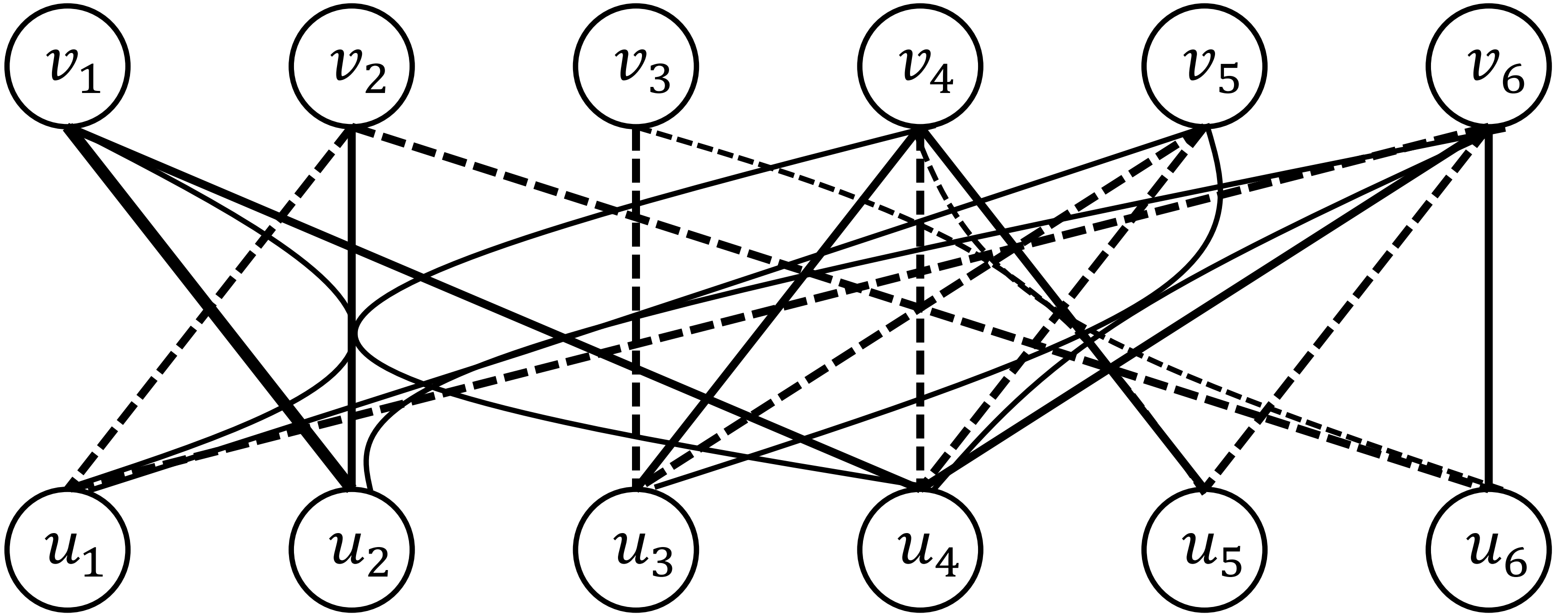
Theorem (B., Heismann [2011])

- Every (hyperedge) clique in a partitioned hypergraph is a subset of the incident hyperedges $\delta(P)$ of some part P.
- The (hyperedge) conflict graph contains no holes of any size and no antiholes of size < 7.



Solution of the LP Relaxation

- Fractional solution, cost = 0.635.
- Consider the 7=2·3+1 cliques associated with the vertices $v_1, v_3, v_4, u_2, u_3, u_4$ and the clique $\{v_5, v_6, u_3, u_4\}, \{v_5, u_3\}, \{v_5, u_4\}$.



Solution of the LP Relaxation

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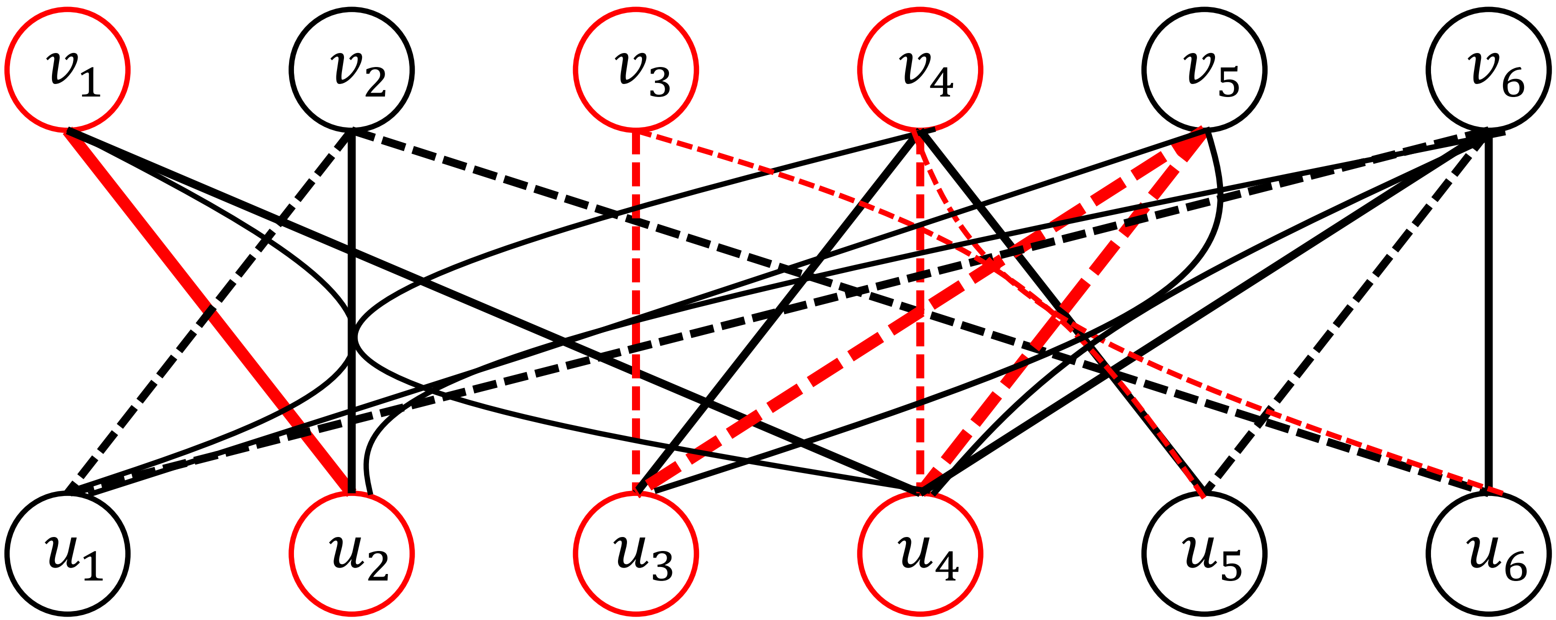
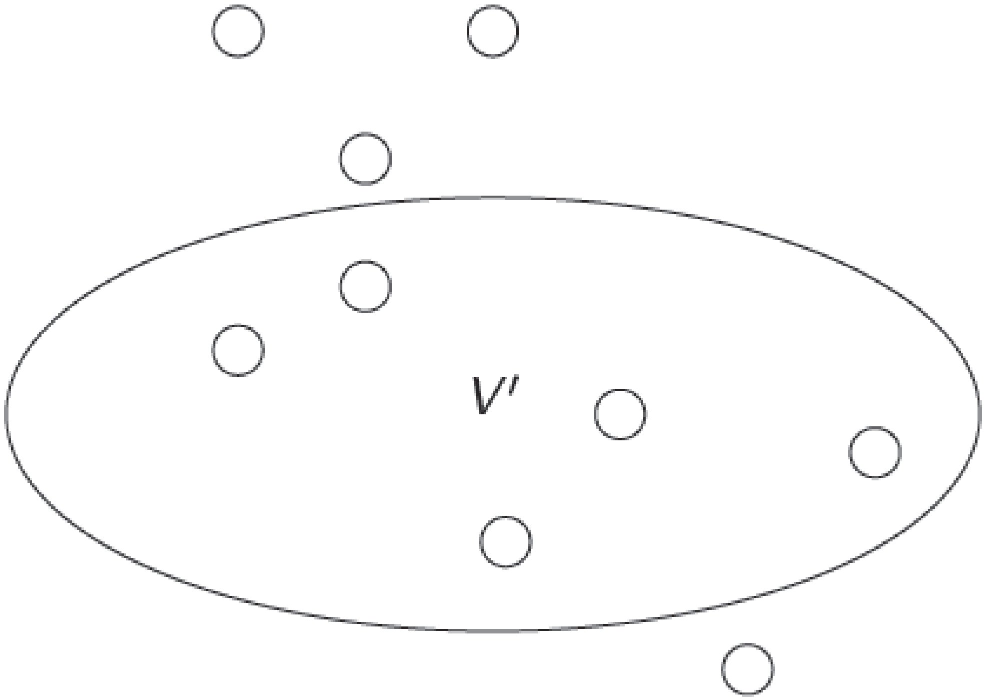
- Every red hyperedge is contained in at least two of these cliques.
- We can take at most three of these edges.



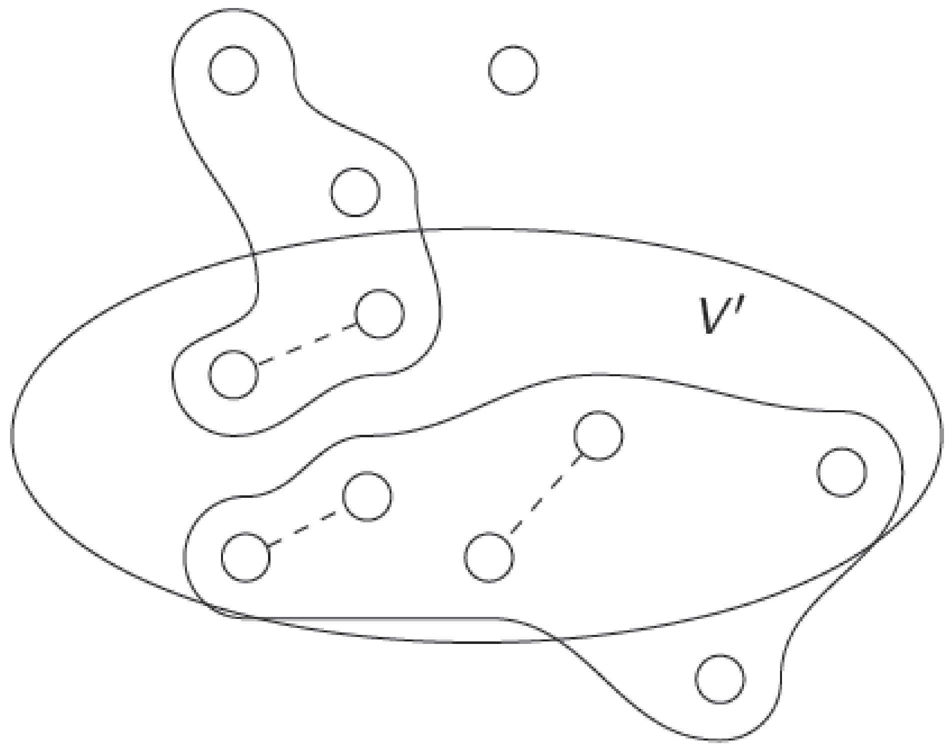
Odd Set Ieqs for the (Perfect) Matching Problem



$$\sum_{e \in E} \left\lfloor \frac{|\{v \in V' : e \in \delta(v)\}|}{2} \right\rfloor x_e \leq \left\lfloor \frac{|V'|}{2} \right\rfloor$$

- Complete description of the matching polytope (together with the degree and non-negativity constraints), Edmonds [1965]

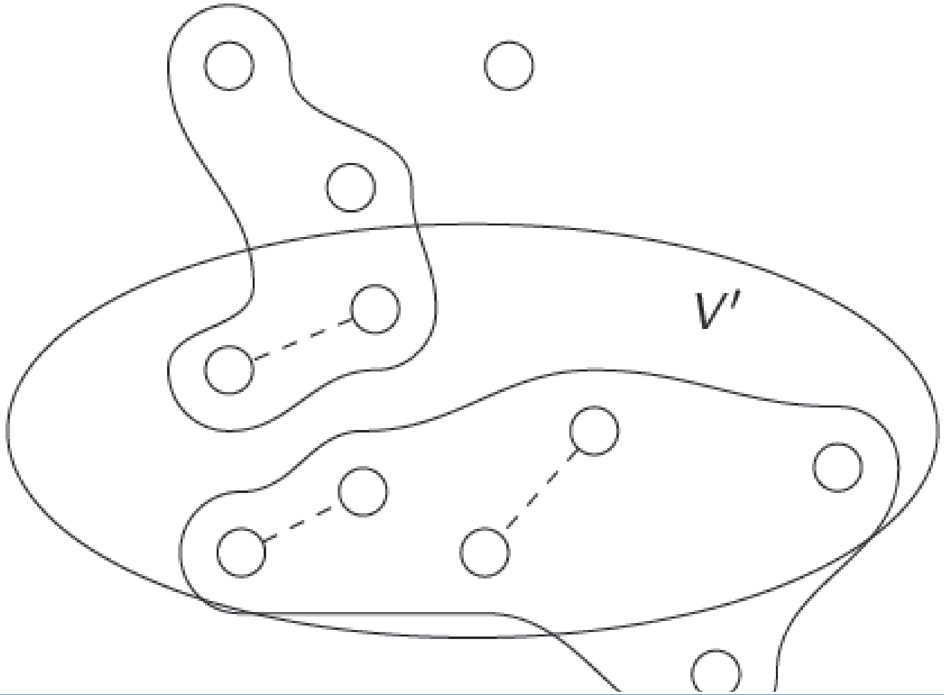
Odd Clique Set Ieqs for General Hypergraphs



$$\sum_{e \in E} \left\lfloor \frac{|\{v \in V' : e \in \delta(v)\}|}{p} \right\rfloor x_e \leq \left\lfloor \frac{|V'|}{p} \right\rfloor \quad \forall V' \subseteq V$$

- Related to clique set inequalities by Pêcher & Wagler [2006]

Odd Clique Set Ieqs for General Hypergraphs



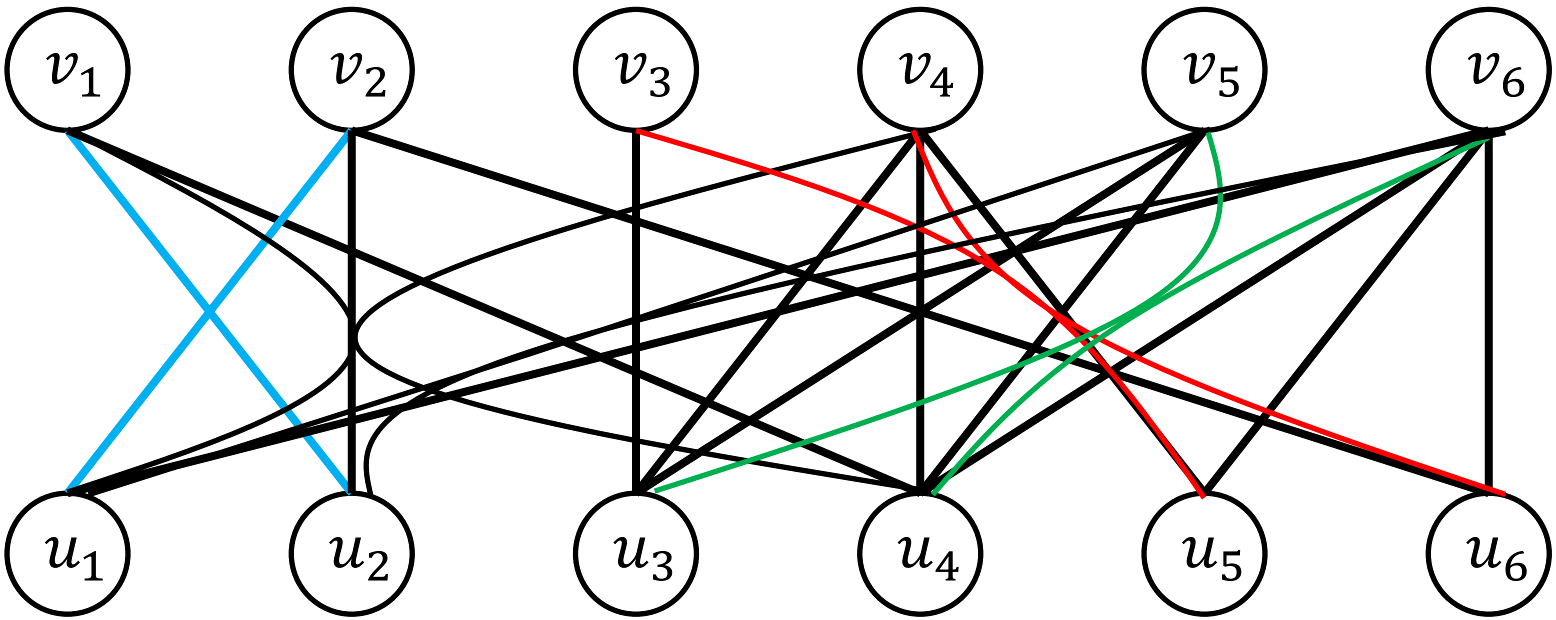
Theorem (B., Heismann [2011])

Let $\boldsymbol{Q}$ be a set of at least three hyperedge cliques in $G = (V, E)$, $2 \leq p \leq |\boldsymbol{Q}|$ be an integer number, $r := |\boldsymbol{Q}| \bmod p$, and $q_e := |\{Q \in \boldsymbol{Q} : Q \ni e\}|$. Then

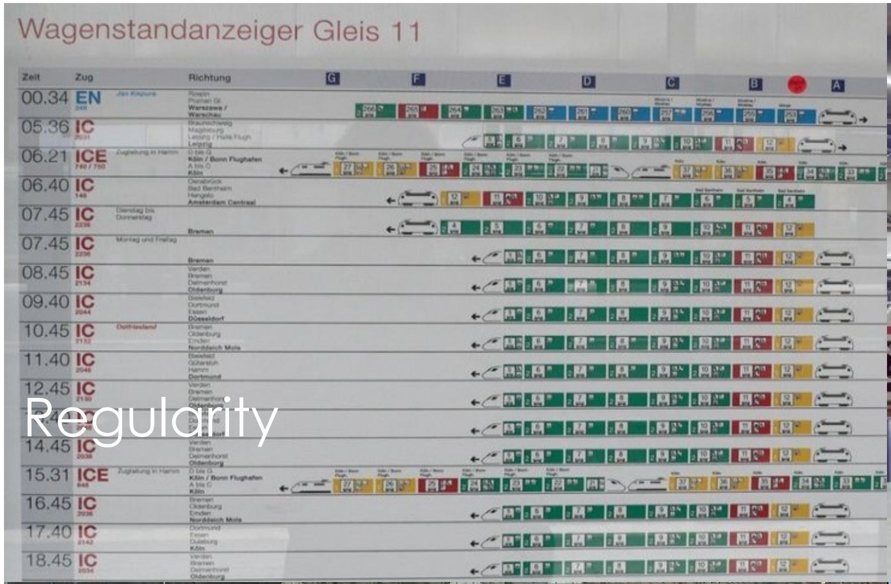
$$\sum_{e \in E} \left(\left\lfloor \frac{q_e}{p} \right\rfloor + \max\left\{0, \frac{q_e \bmod p - r}{p - r}\right\} \right) x_e \leq \left\lfloor \frac{|\boldsymbol{Q}|}{p} \right\rfloor.$$

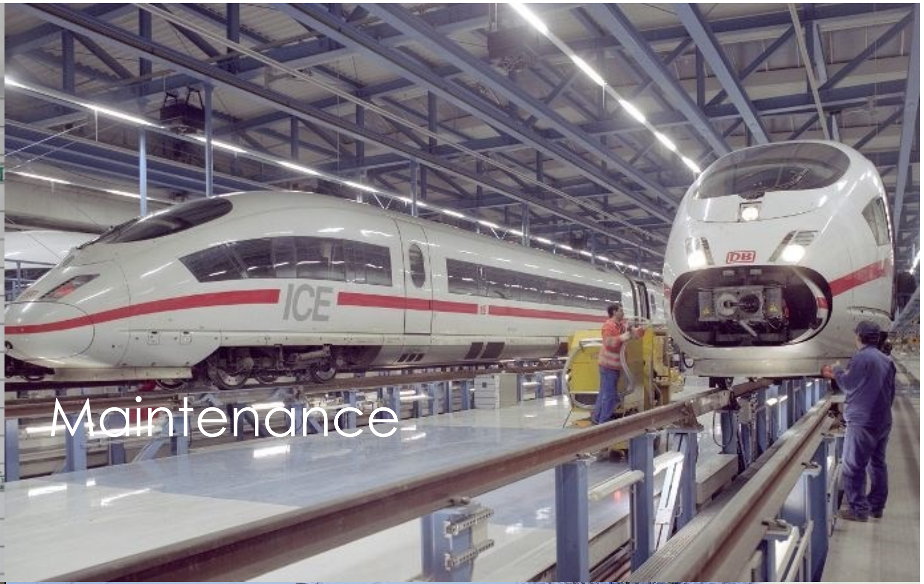
Hyperassignment Solution

- Integer solution, cost = 1.010.



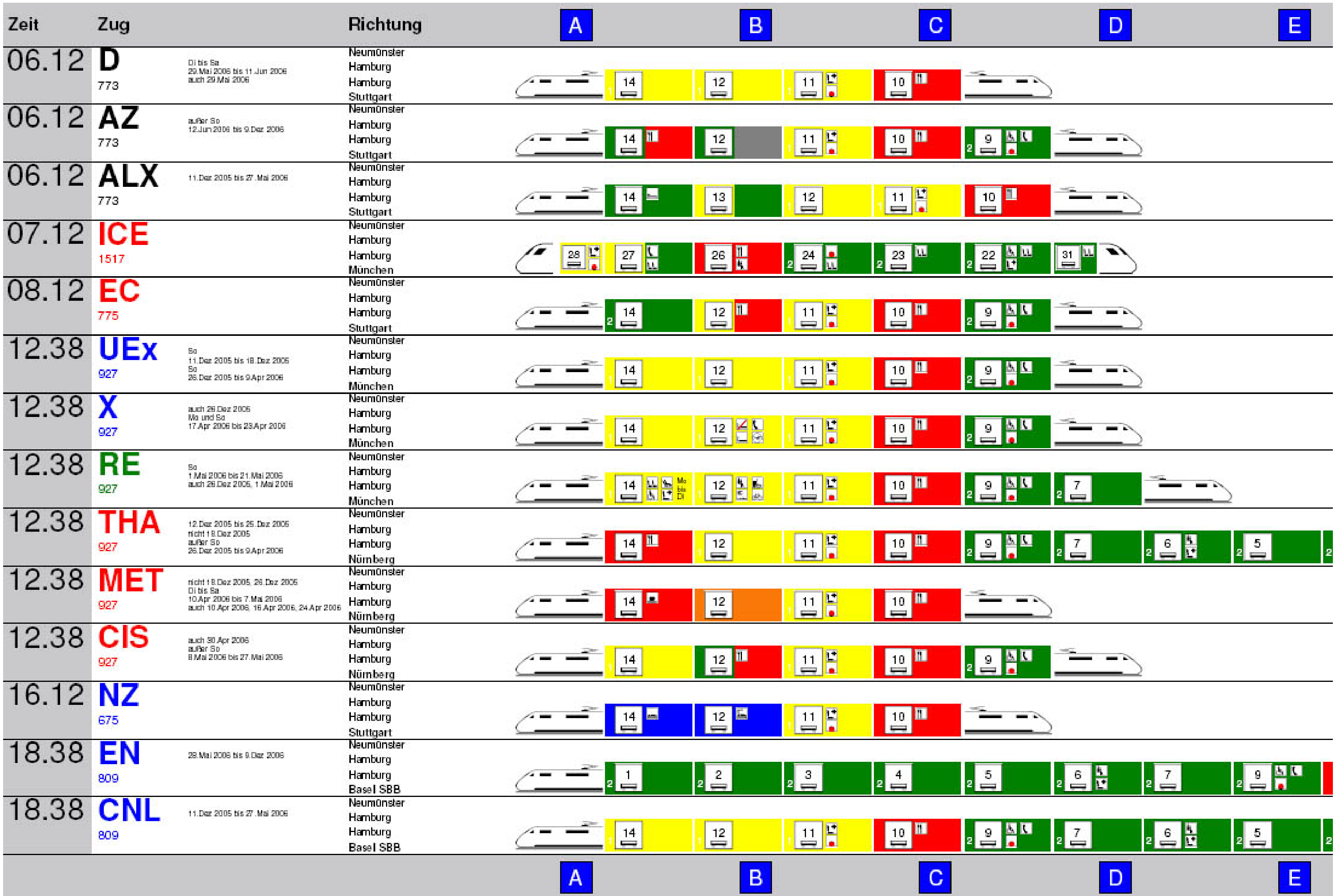
Railway Constraints





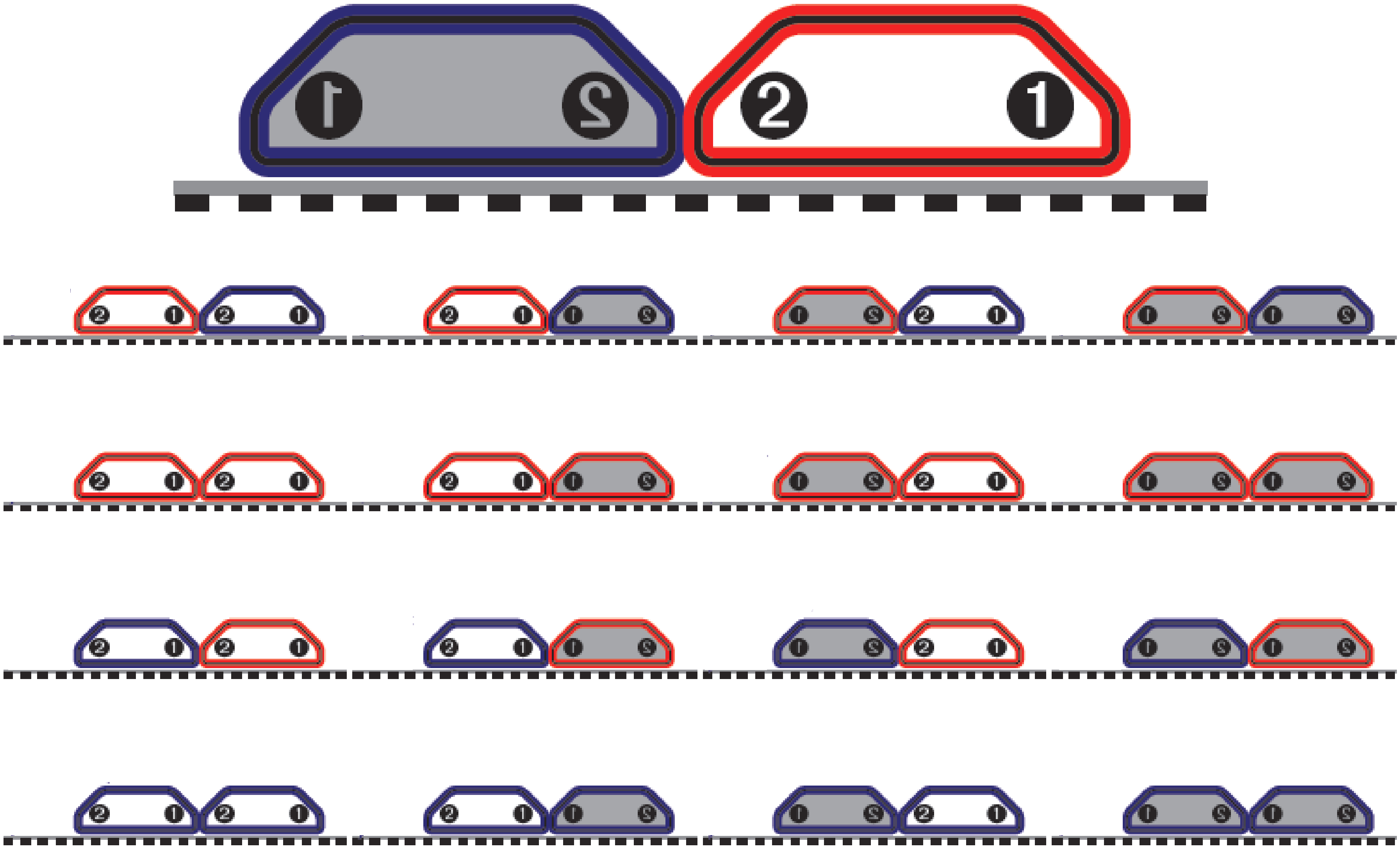
Photos courtesy of DB Mobility Logistics AG

Train Composition

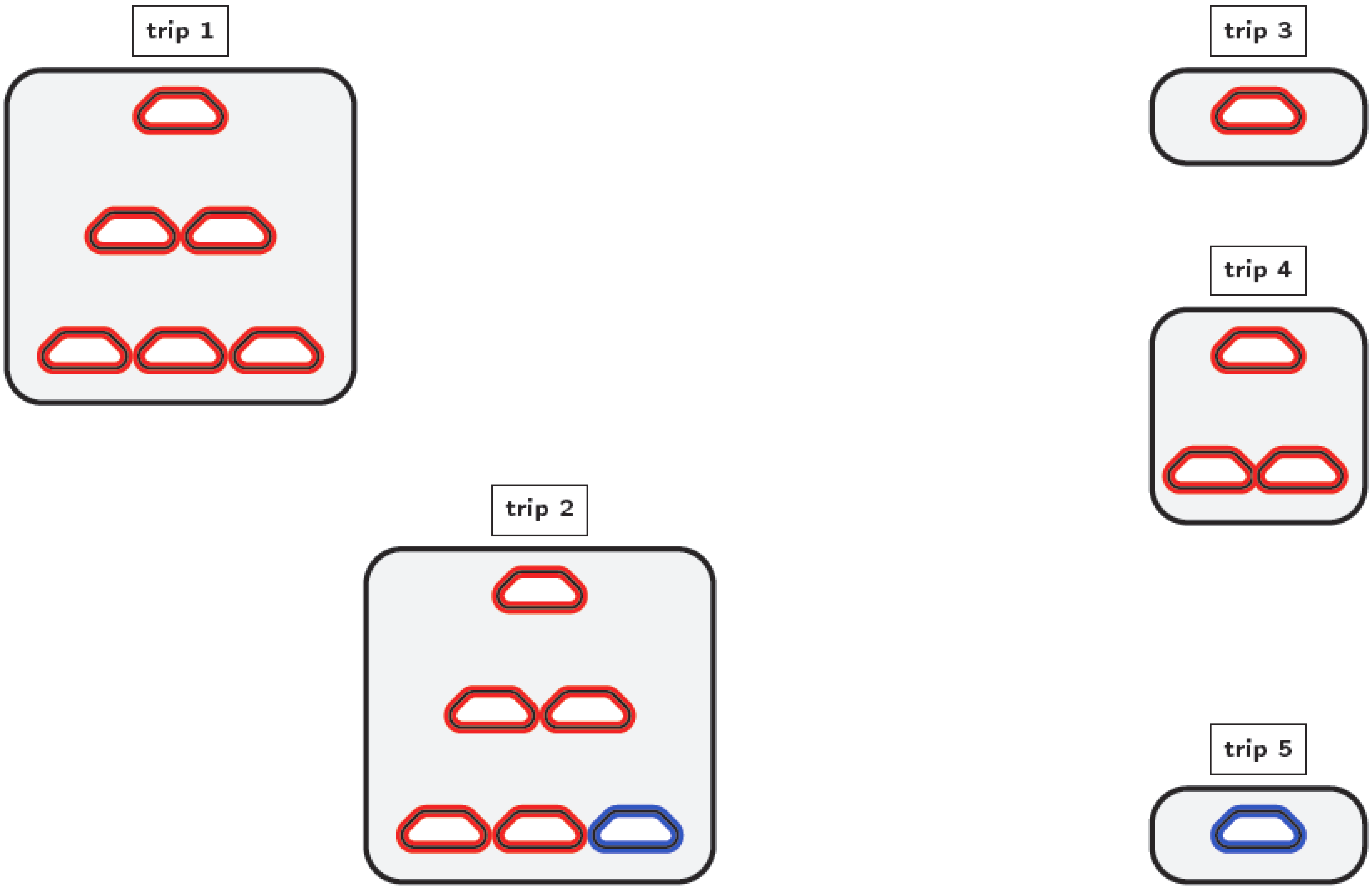


Rare Train Composition Example

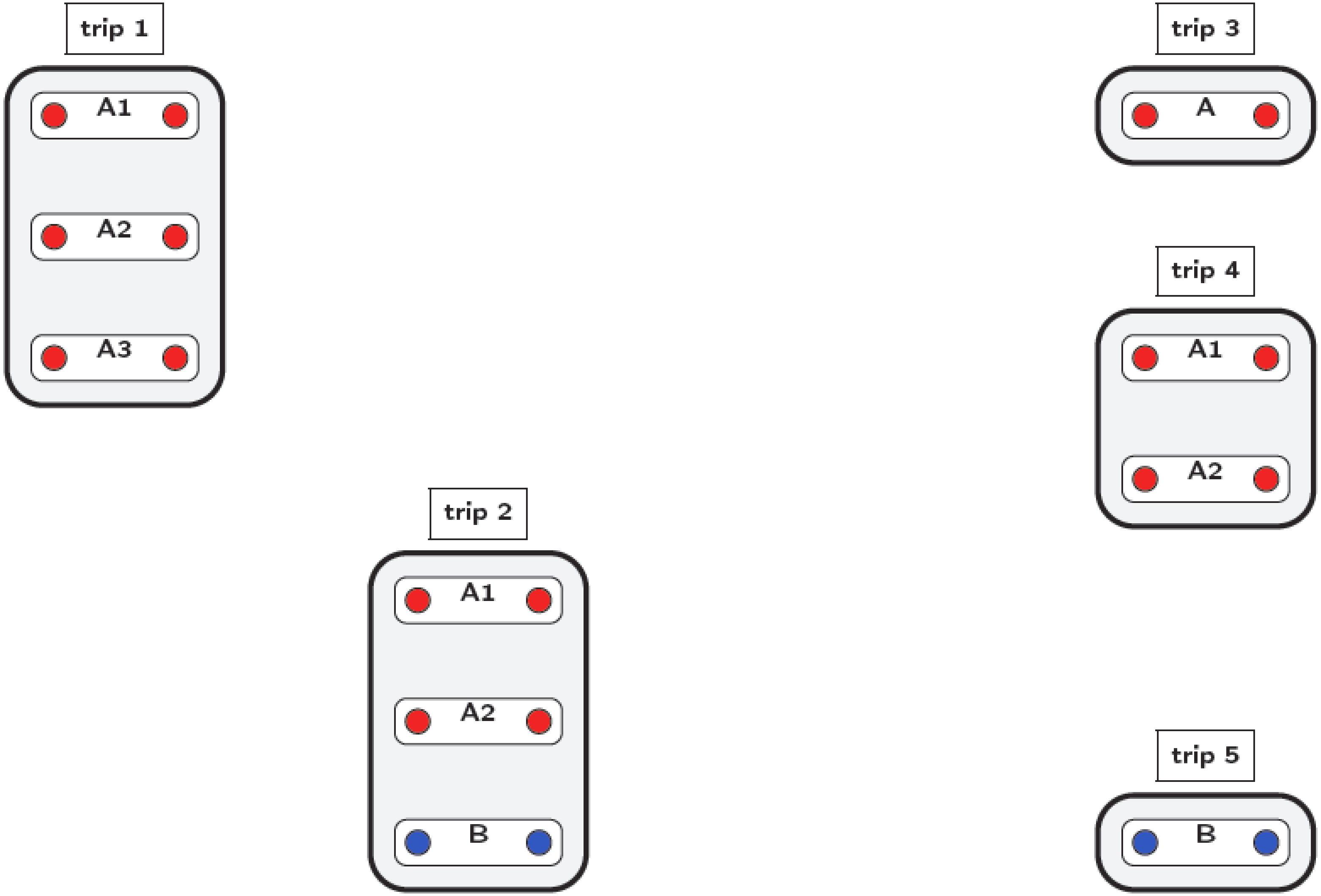
Train Composition: Type, Order, Orientation



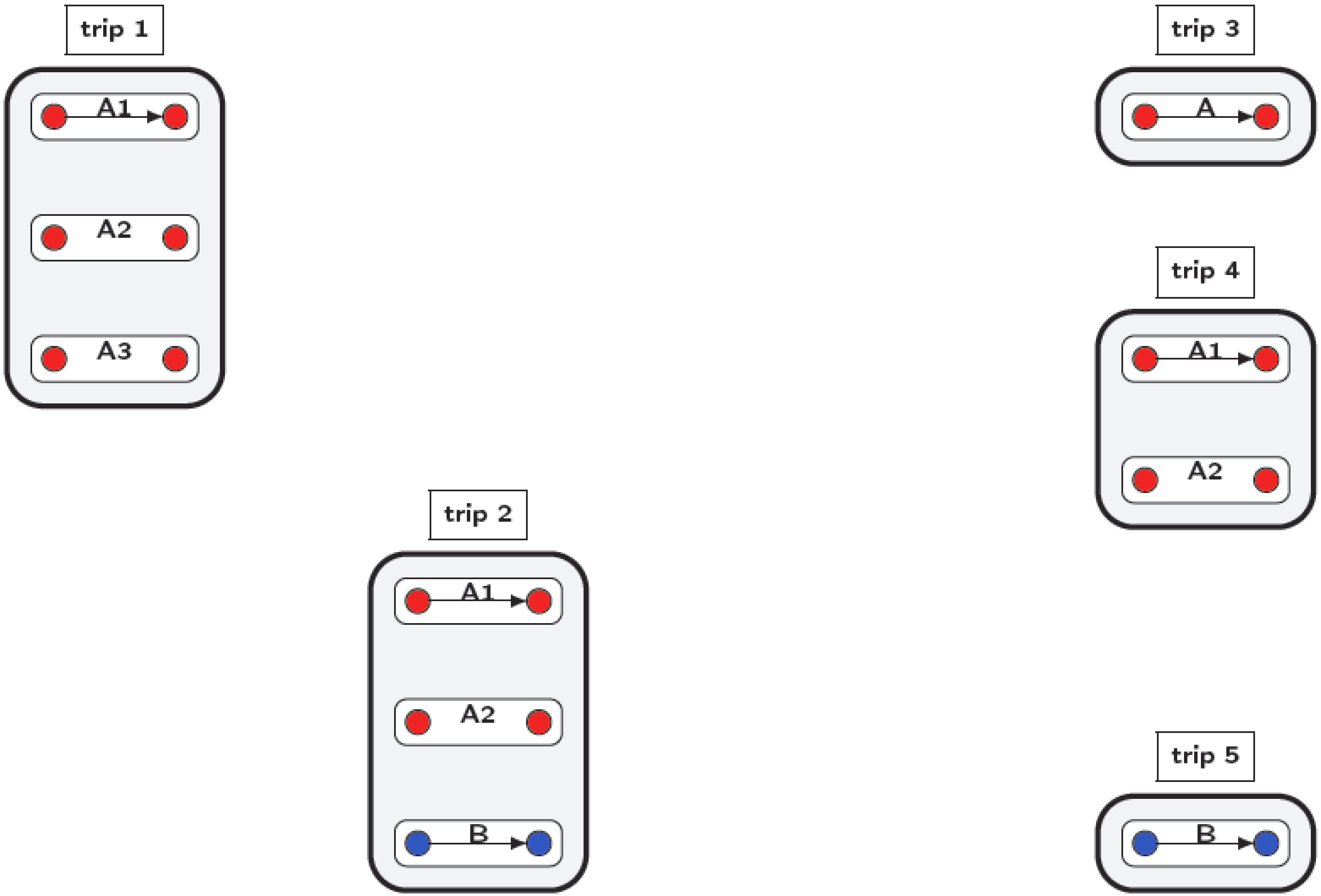
Hypergraph Model: Possible Train Compositions



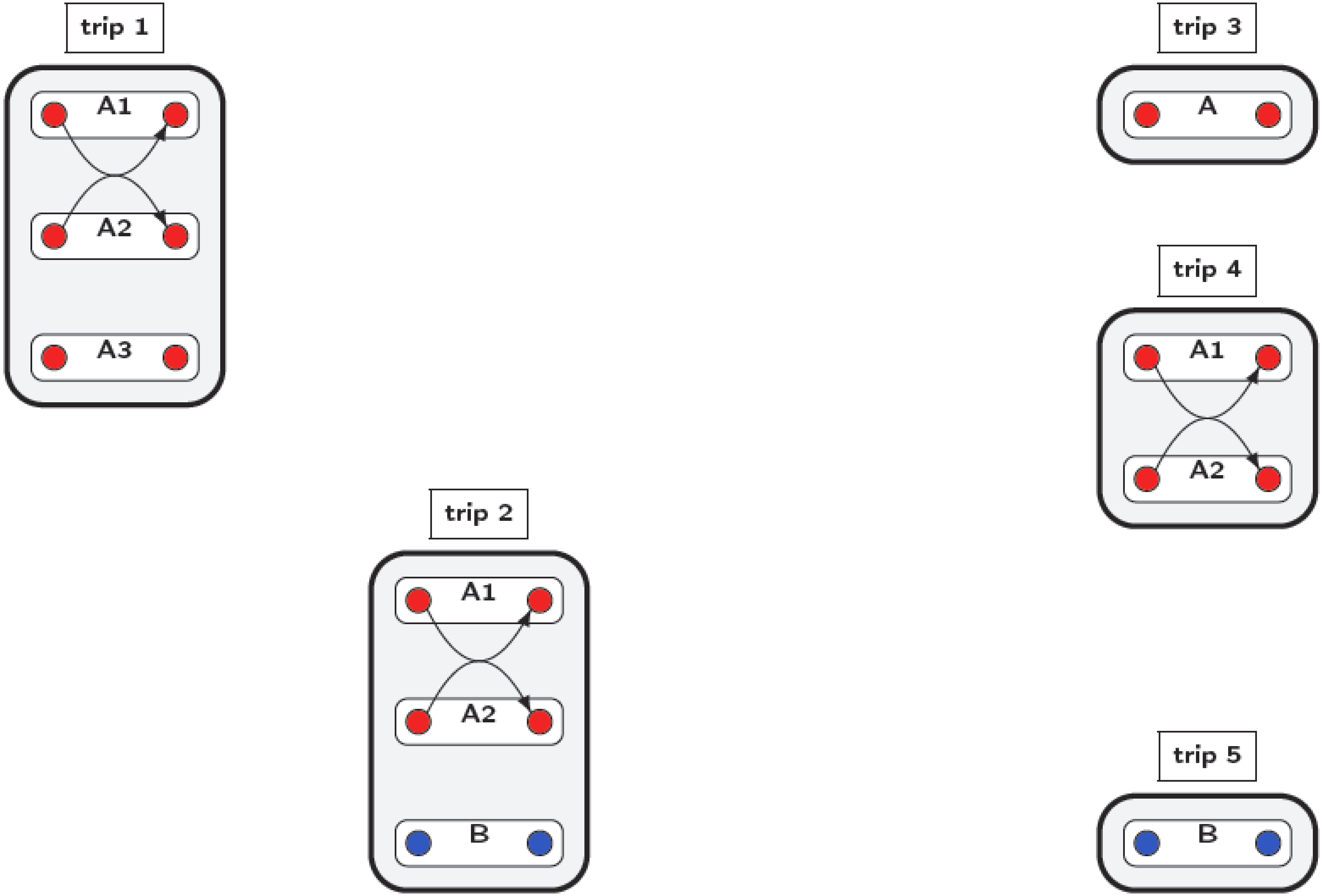
Hypergraph Model: Arrival and Departure Nodes



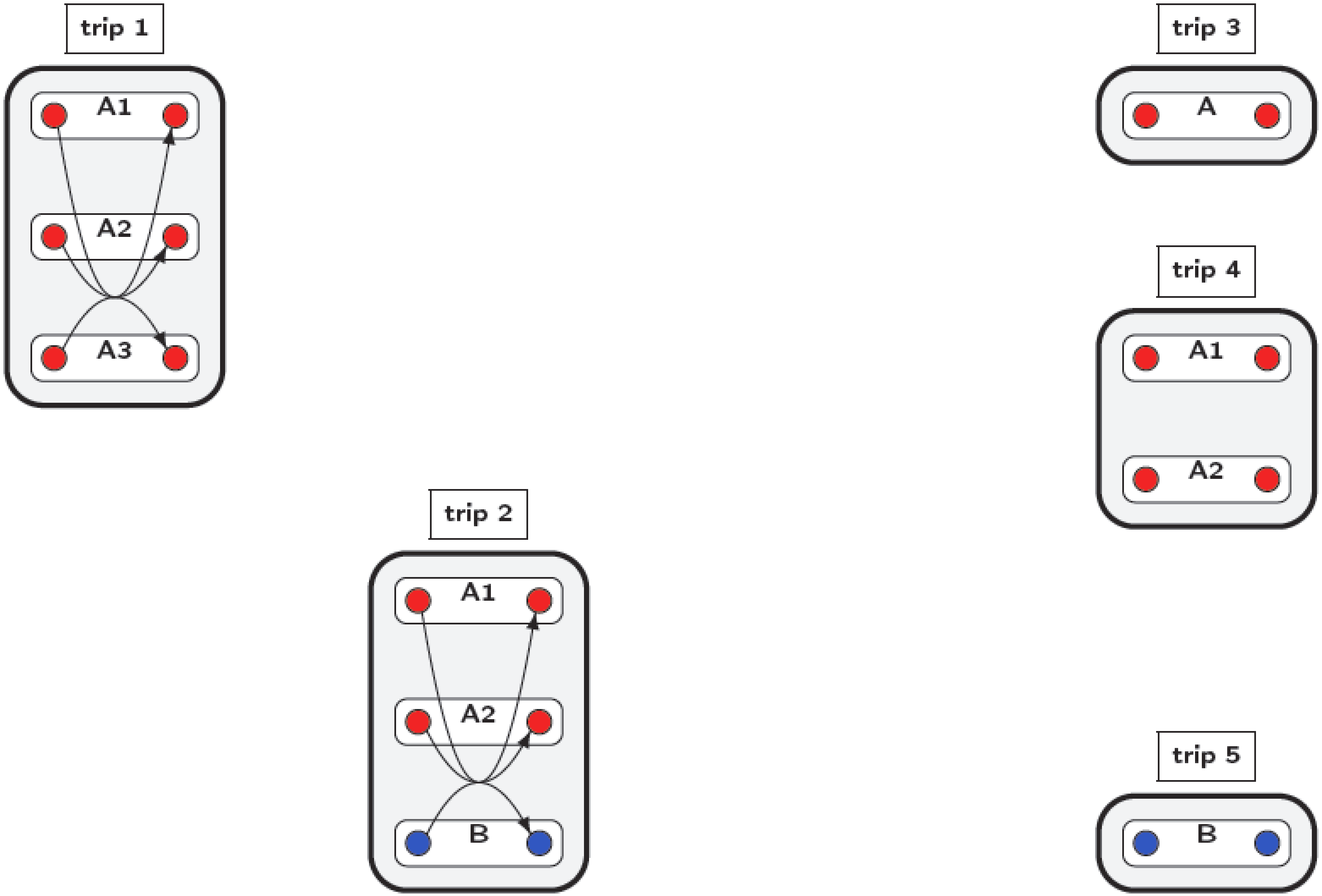
Hypergraph Model: Single Traction



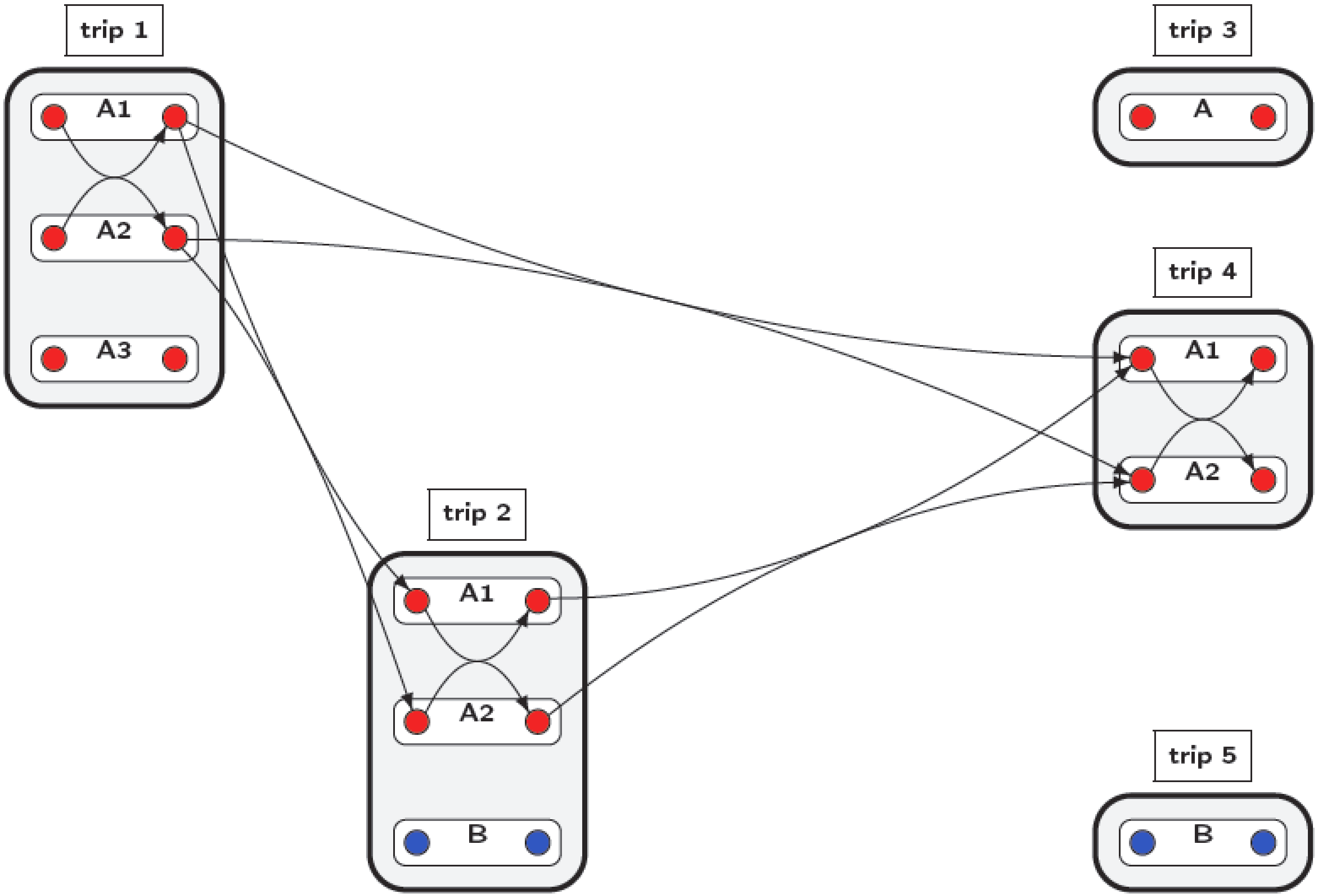
Hypergraph Model: Double Traction



Hypergraph Model: Triple Traction



Hypergraph Model: Pass-Through Connections



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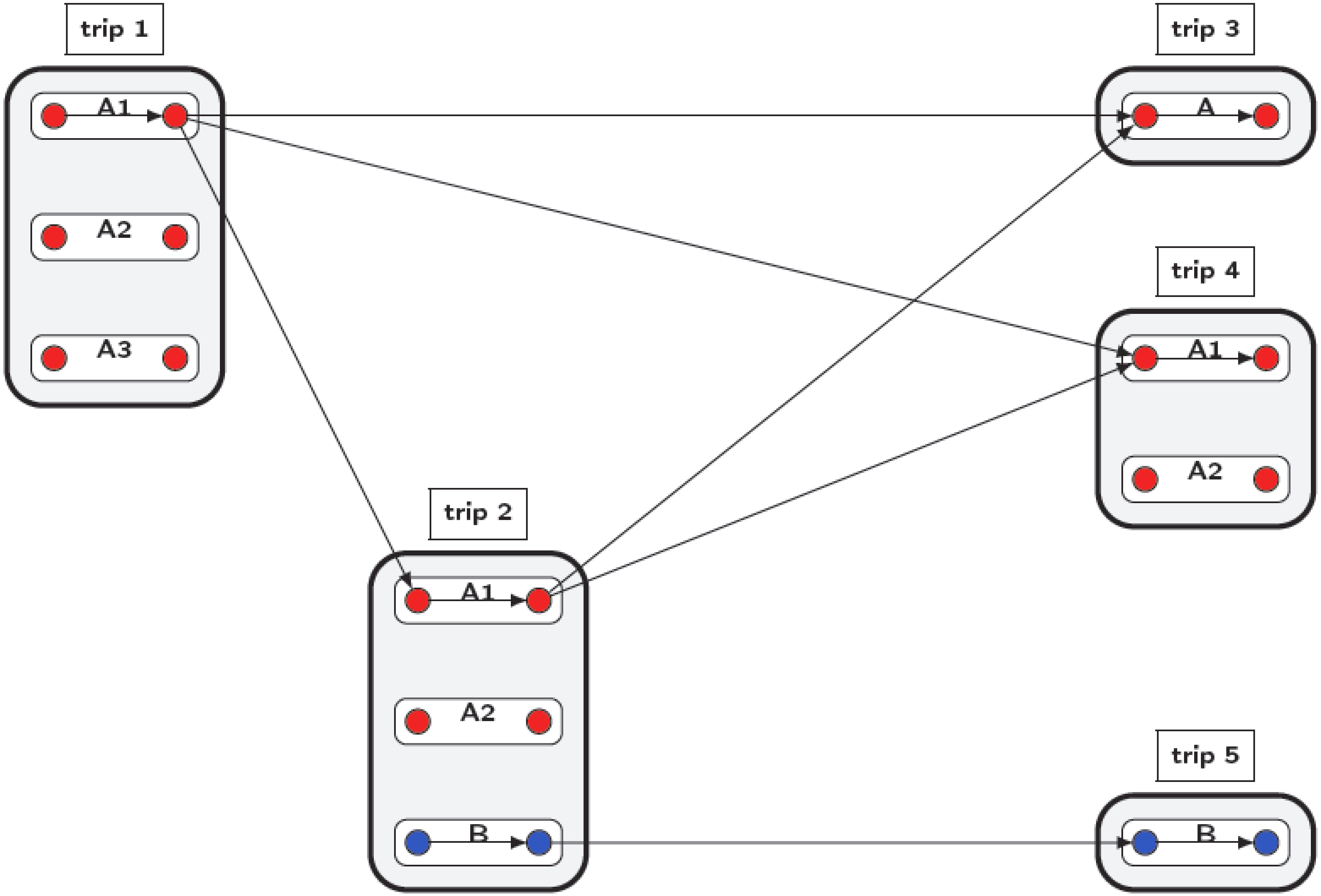
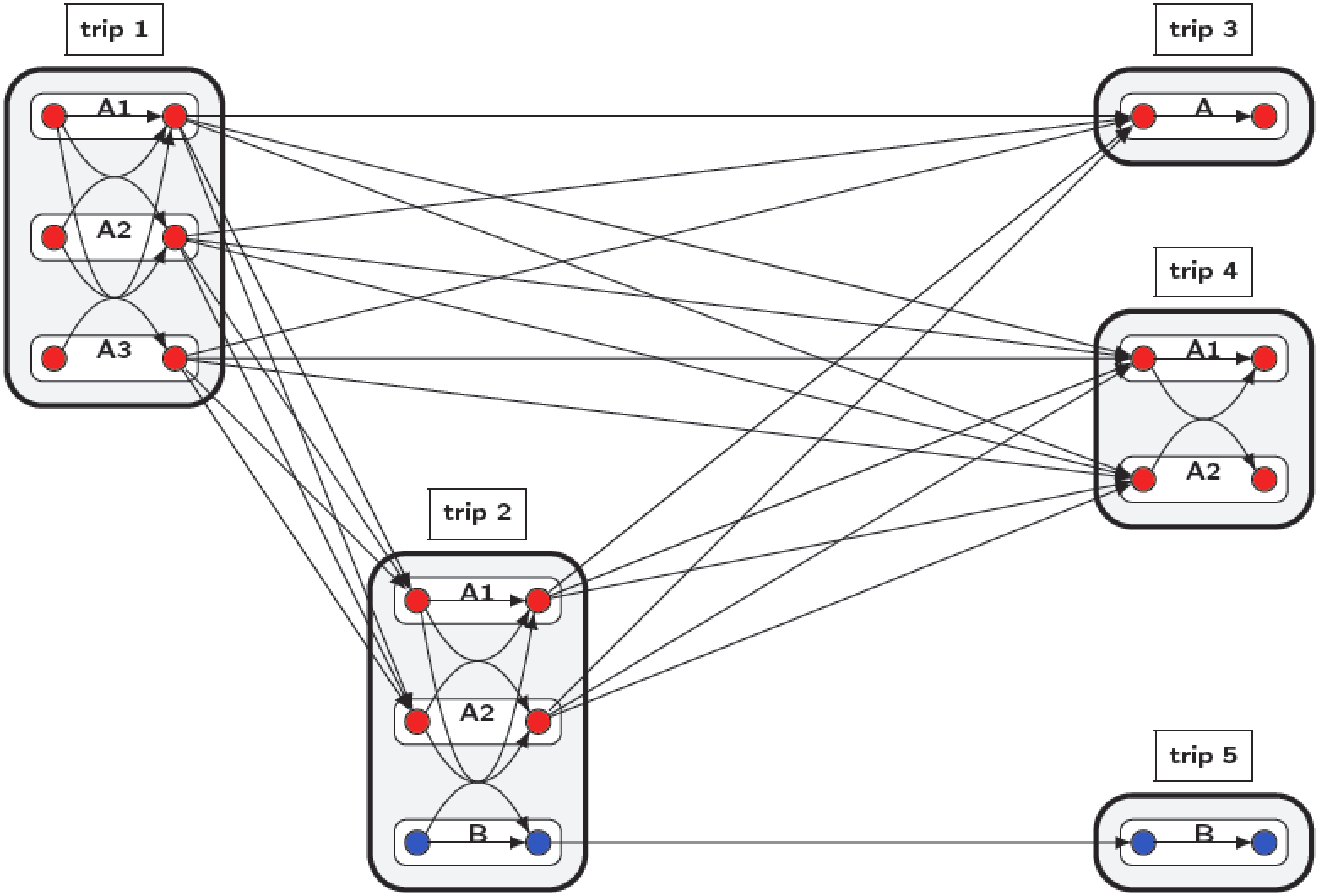
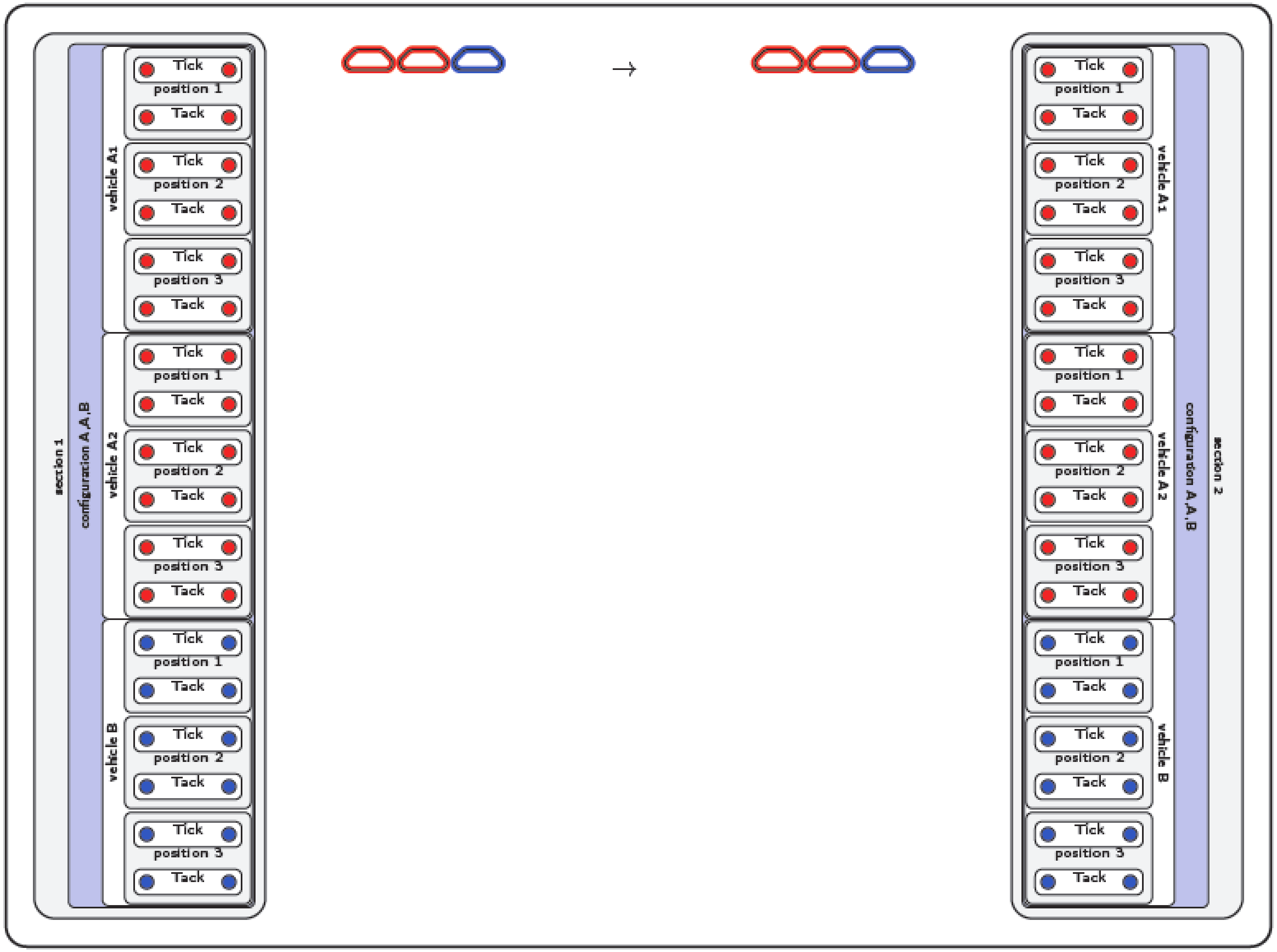


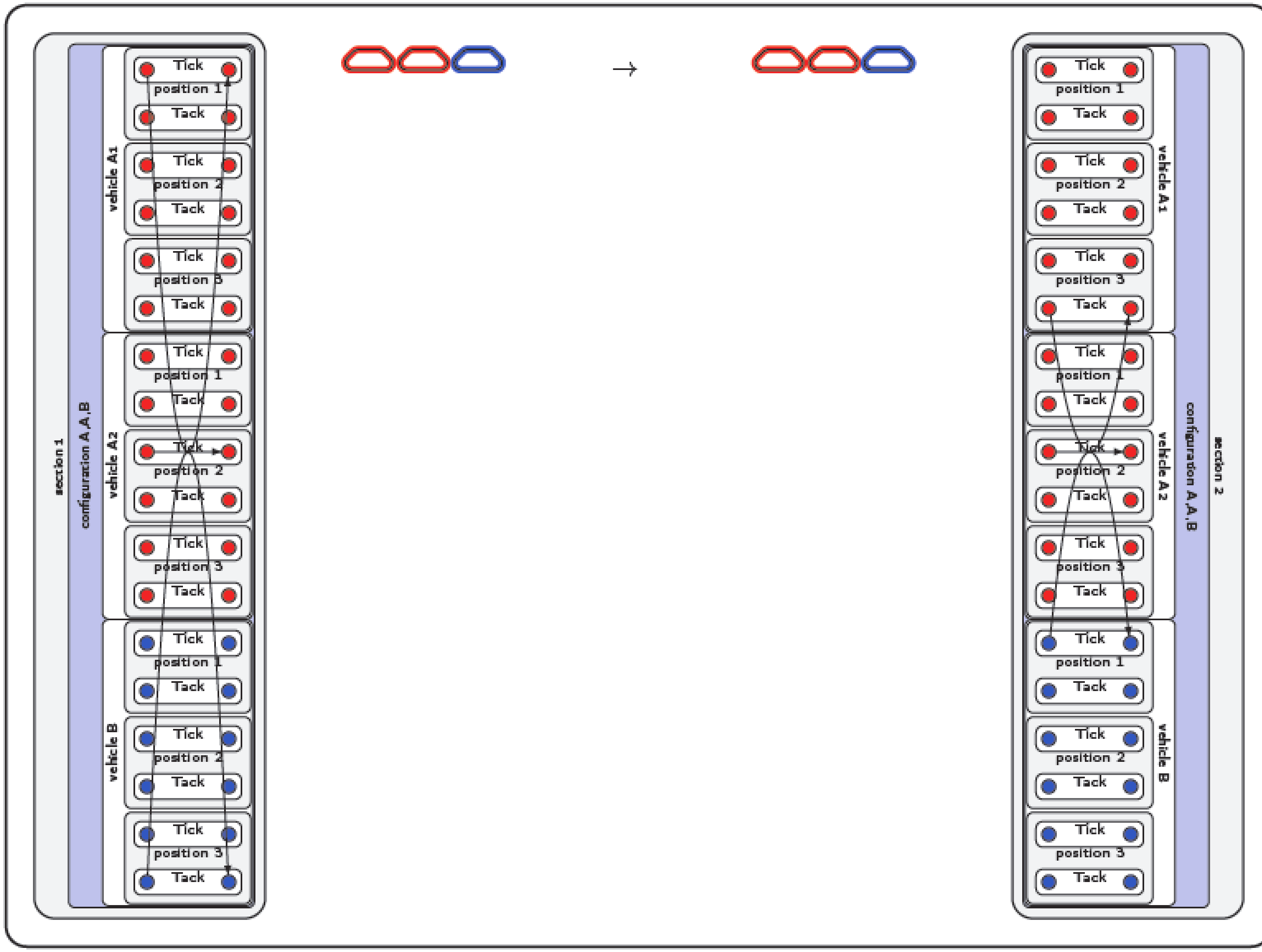
Hypergraph Model: All Connections



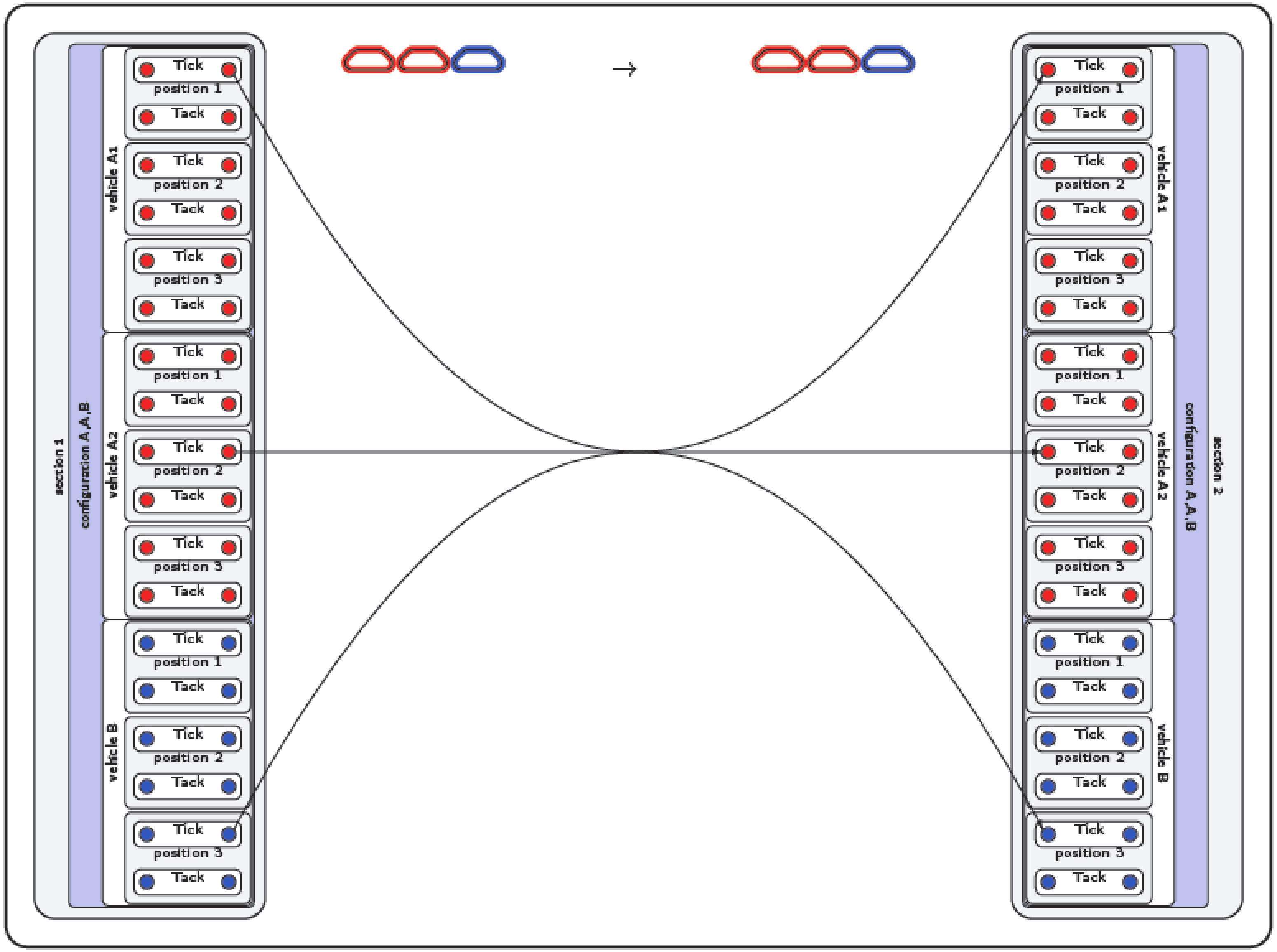
Hypergraph Model: Zoom



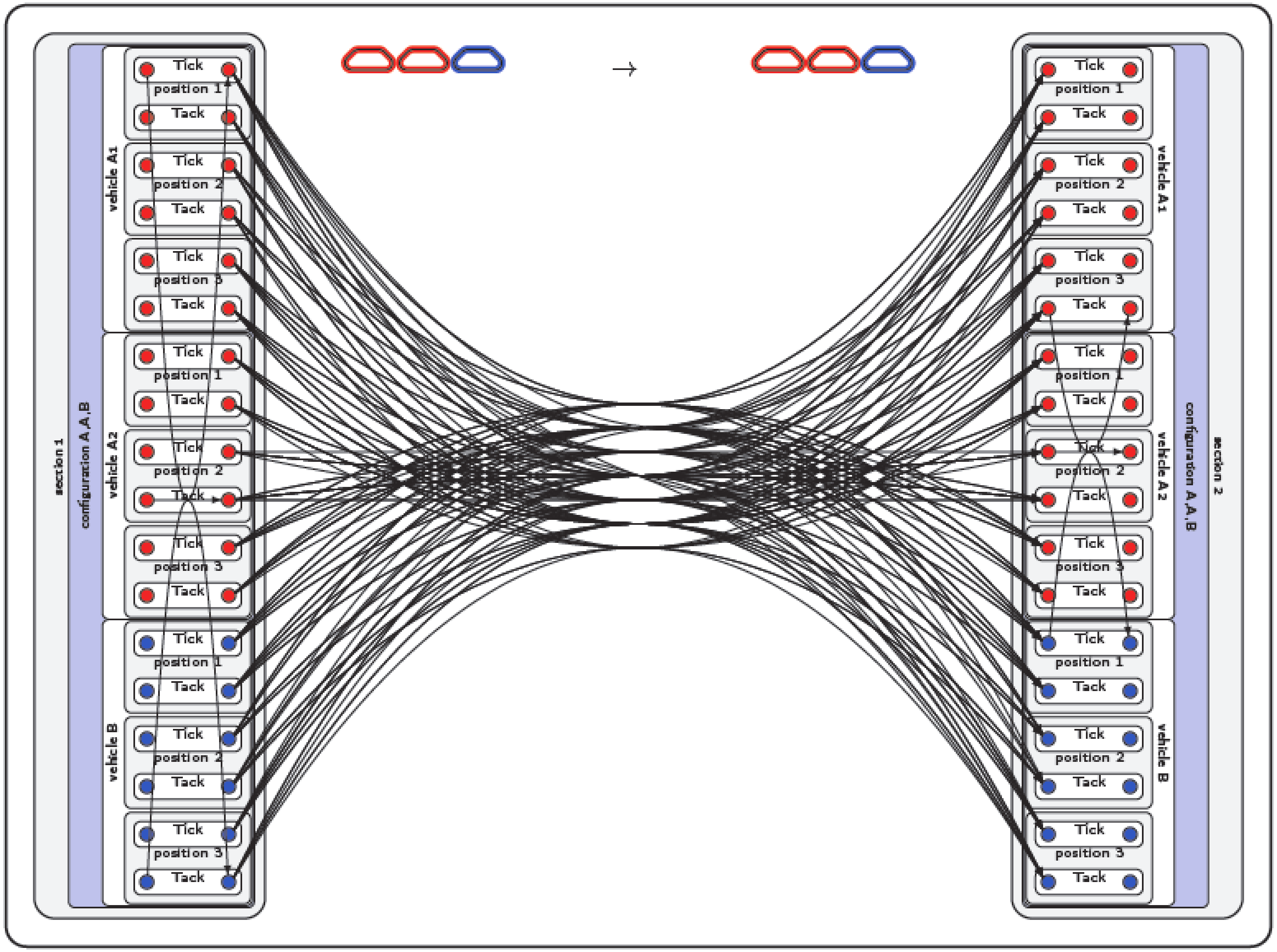
Hypergraph Model: Timetabled Trips



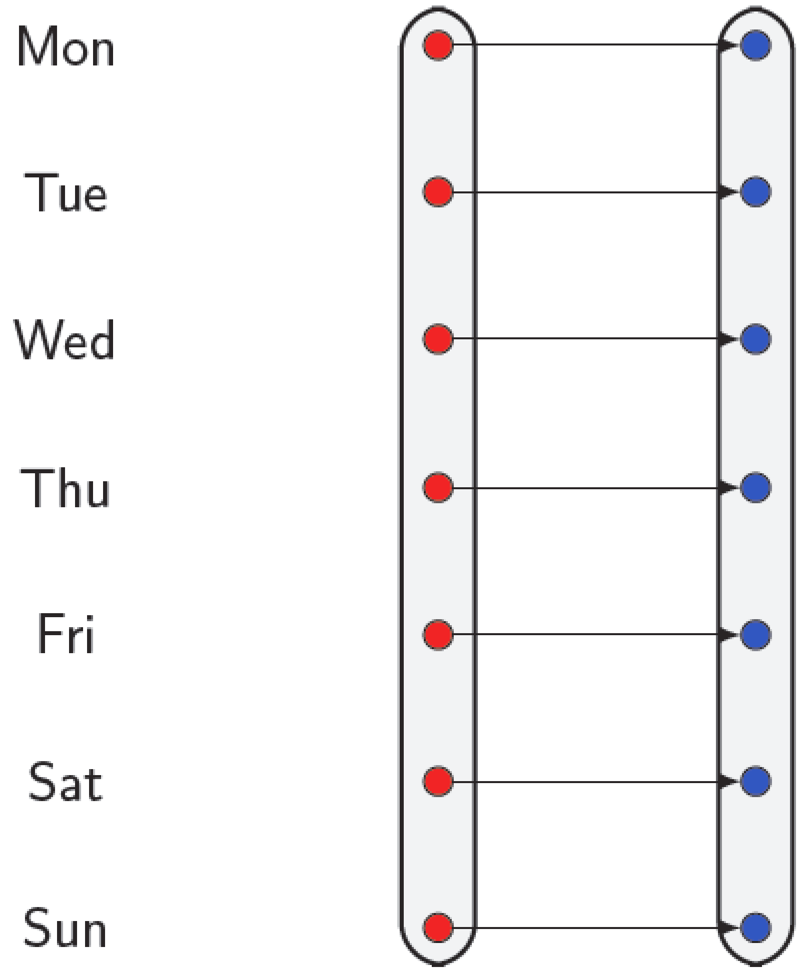
Hypergraph Model: Connections



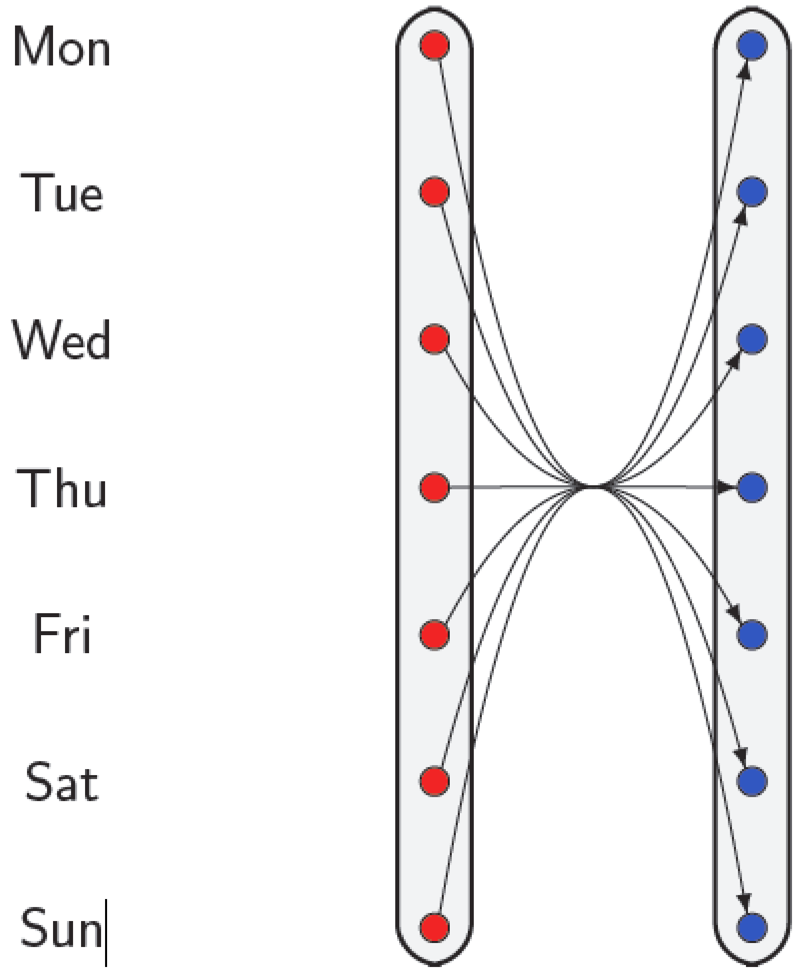
Hypergraph Model: All Connections



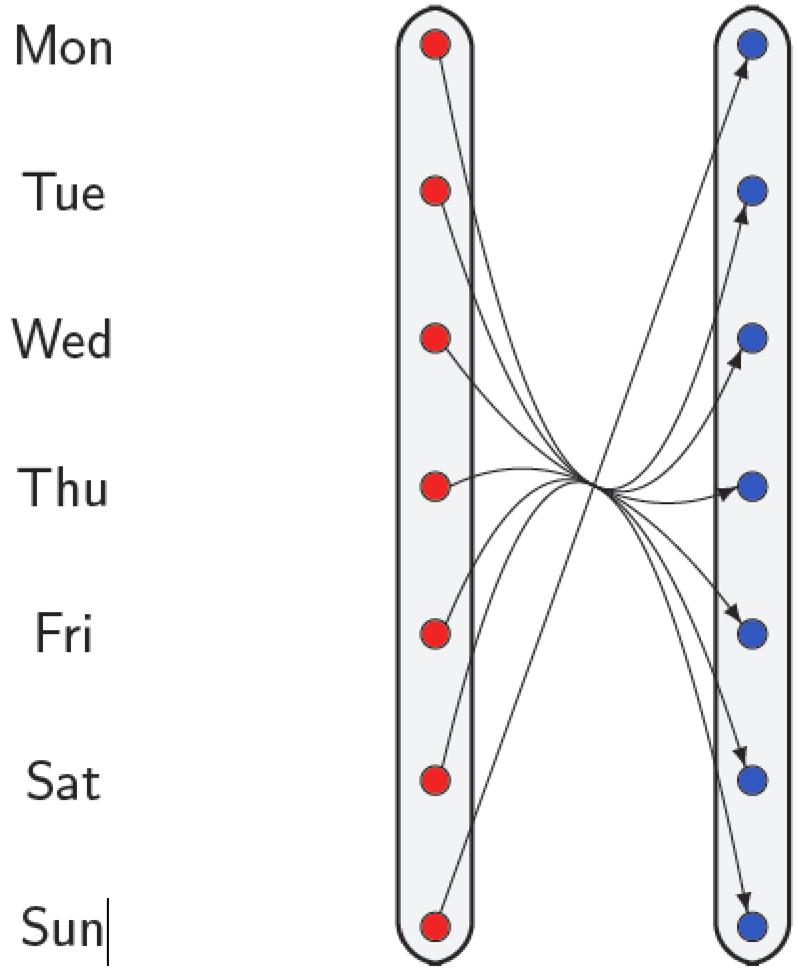
Hypergraph Model: Two Daily Trains



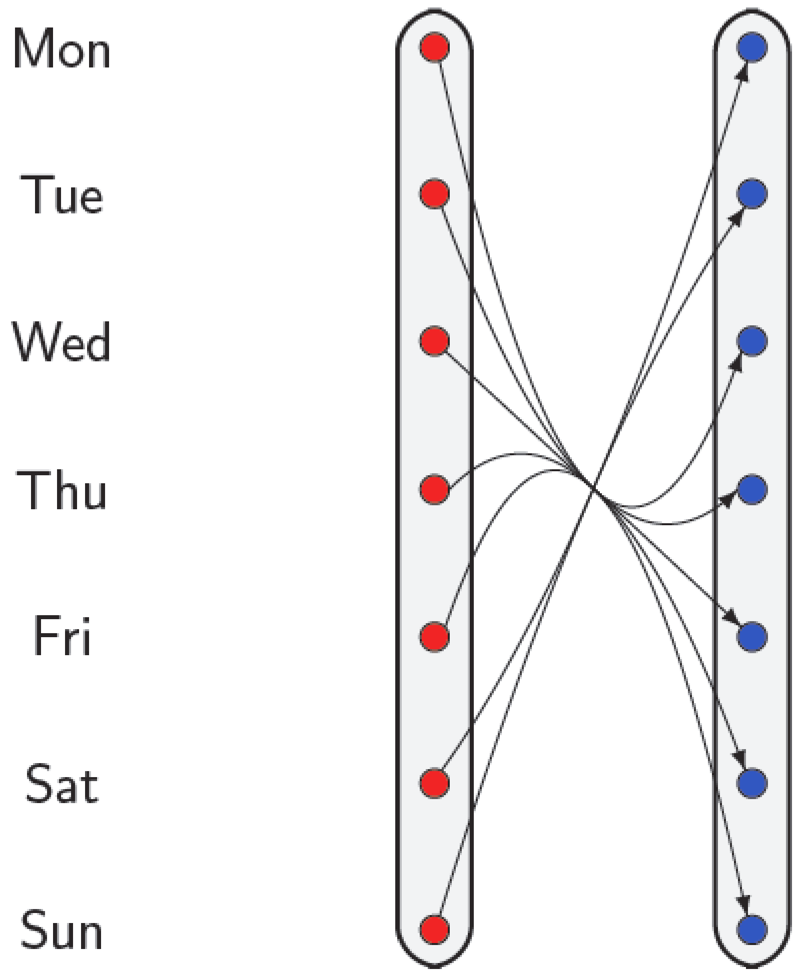
Hypergraph Model: Max Regularity (0 Idle Days)



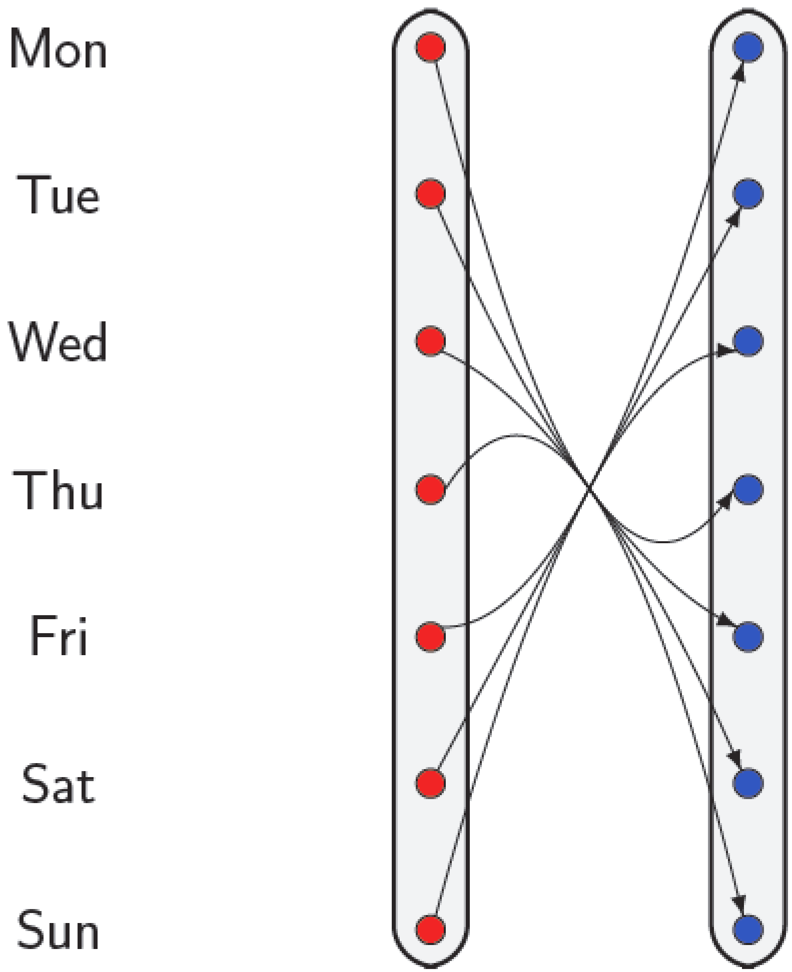
Hypergraph Model: Max Regularity (1 Idle Day)



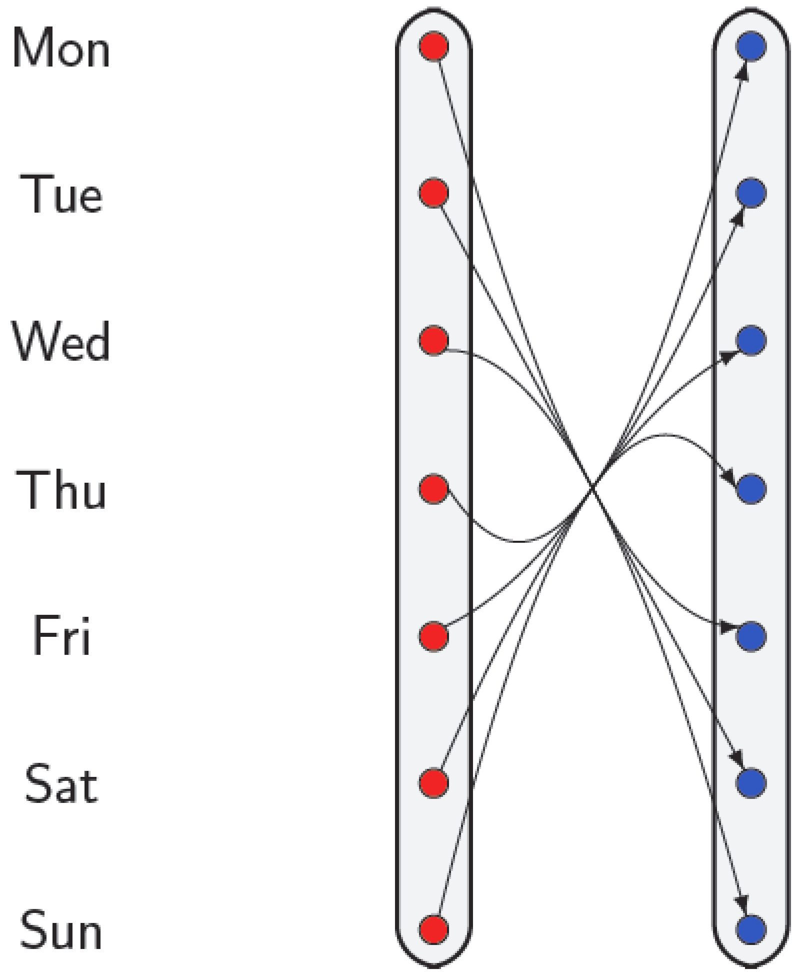
Hypergraph Model: Max Regularity (2 Idle Days)



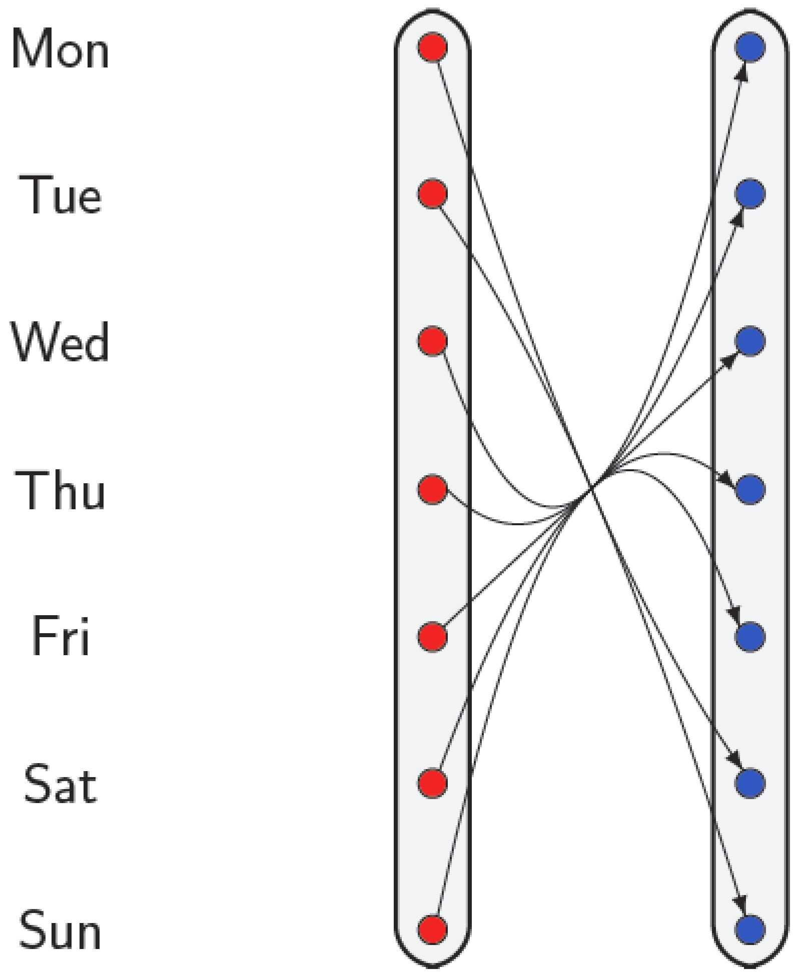
Hypergraph Model: Max Regularity (3 Idle Days)



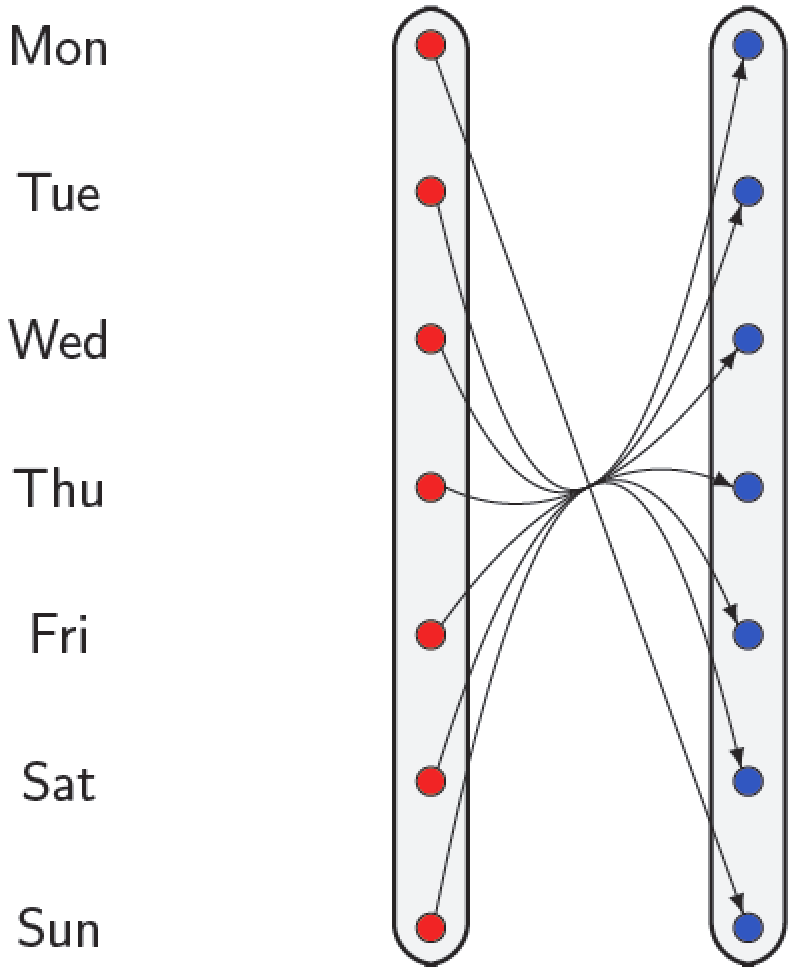
Hypergraph Model: Max Regularity (4 Idle Days)



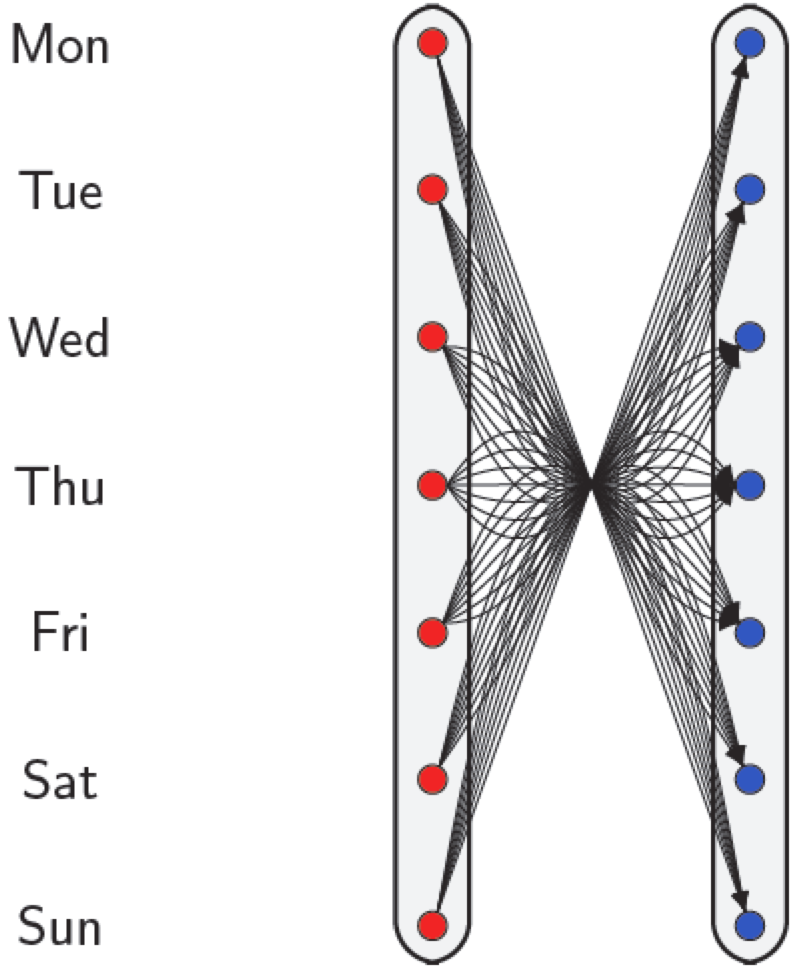
Hypergraph Model: Max Regularity (5 Idle Days)



Hypergraph Model: Max Regularity (6 Idle Days)



Hypergraph Model: Max Regularity (All Options)



Hyperflow Model

Vehicle Rotation Planning Problem

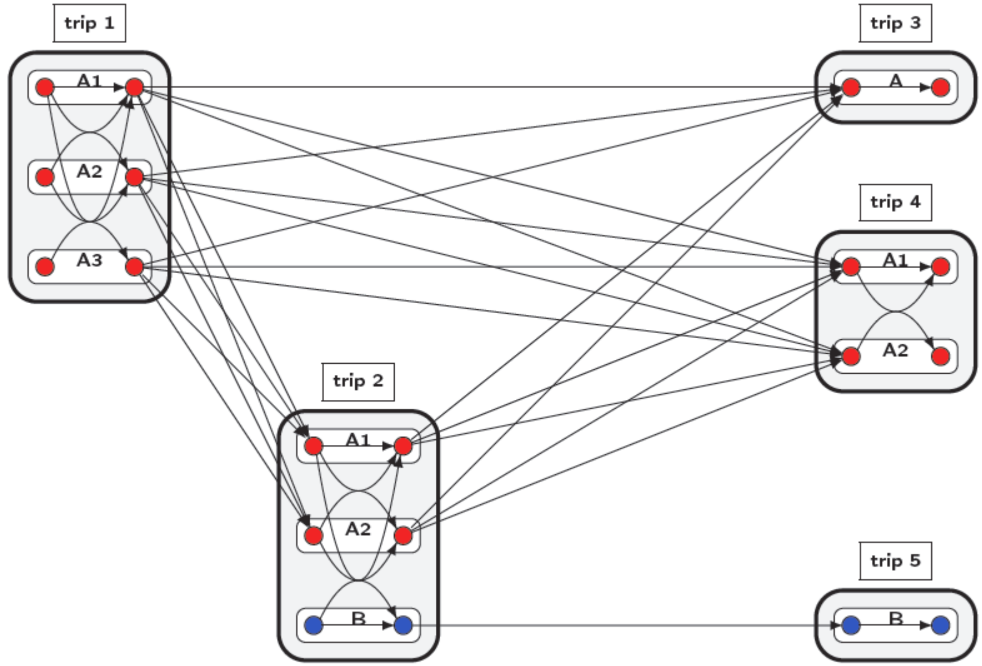
Cover all timetabled trips by rotations such that turns and train composition are regular.

Hypergraph Multi Commodity Flow Problem

Find a cost minimal hyperflow such that every node configuration is covered by exactly one hyperarc.

Hyperarcs for

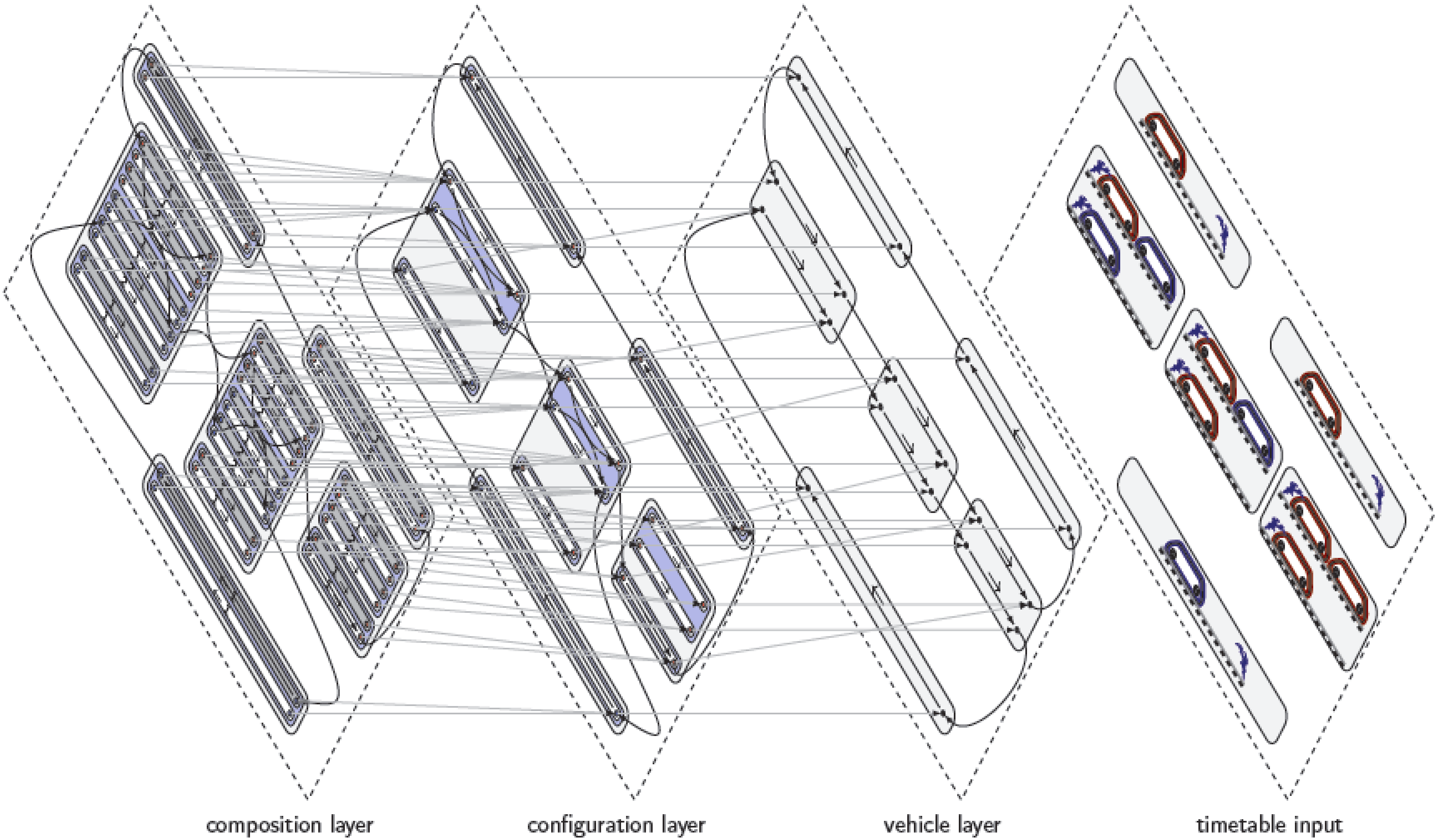
- (regular) trips
- (regular) connections



$$\min c^T x$$

$$\begin{aligned} x(d,\delta^+(v)) &= x(d,\delta^-(v)) && \forall v \in V, d \in D \\ x(\delta^+(t)) &= 1 && \forall t \in T \\ x &\in \{0,1\}^{A \cup M} \end{aligned}$$

The Coarse-to-Fine Method



Coarse-to-Fine Method: Layers

Problem specific layers:

- composition layer (with order and orientation, fine)
- configuration layer (with types and combinations, coarse)
- vehicle layer (individual vehicle flow, very coarse)

The layers are defined in terms of projections of hypergraphs that correspond to the projection of rows of the LP/IP.

Coarse-to-Fine Method: General Setting

- Row and column index sets $I = [m], J = [n]$
- Matrix $A \in \mathbb{R}^{I \times J}$
- Rhs $b \in \mathbb{R}^{I}$
- Objective $\mathrm{c} \in \mathbb{R}^{J}$
- Linear Program and its dual

$$\begin{aligned} \min c^T x & \\ Ax &= b \\ x &\geq 0 \end{aligned}$$

$$\begin{aligned} \max\ y^T b & \\ y^T A &= c^T \\ y &\in \mathrm{IR}^I \end{aligned}$$

Coarse-to-Fine Method: Notation

- Aggregate/project the rows I of the (LP) by a problem specific coarsening projection $[]: I \to [I]$ (induces an equivalence relation)

- For a column vector $v \in \mathbb{R}^I$ we define the coarsening of v as

$$[v][i] \coloneqq (\min\{v_k : k \in I, [k] = [i]\}, \max\{v_k : k \in I, [k] = [i]\}) \cdot \tau(v, i)$$

where $\tau(v, i) \coloneqq |\{v_k \neq 0 : [k] = [i]\}|$

- Coarse bimatrix $[A]$, coarse dual vector $[\pi]$

- Coarse objective function $[c] \coloneqq c$ (no coarsening)

Coarse-to-Fine Method: Coarse Reduction

$$\min [c]^T x, [A]x[=][b], x \in \mathbb{R}^{[J]},$$

where

$$[A]x[=][b] :\Leftrightarrow [b]_{[i]1} \leq \sum_{j\in J} [A_{\cdot j}]_{[i]2} x_j,\ \sum_{j\in J} [A_{\cdot j}]_{[i]1} x_j \leq [b]_{[i]2},\ \forall [i].$$

Let $P(A,b) \coloneqq \{Ax = b, x \geq 0\}$ and $P([A],[b]) \coloneqq \{[A]x[=][b], x \geq 0\}$.

Lemma (B., Reuther, Schlechte, Weider [2015])

$$P(A,b) \subseteq P([A],[b]).$$

Coarse-to-Fine Method: Coarse Reduced Cost

- Multiplication of pairs $(a_1, b_1), (a_2, b_2) \in \mathbb{R}^2$:

$$(a_1, b_1), \cdot (a_2, b_2) \coloneqq \max\{a_1 b_1, a_1 b_2, a_2 b_1, a_2 b_2\}$$

- Coarse reduced cost for column j

$$[\overline{c}_j] \coloneqq [c_j] - [\pi]^T \cdot [a_j]$$

Lemma (B., Reuther, Schlechte, Weider [2015])

The coarse reduced cost always underestimates the original reduced cost

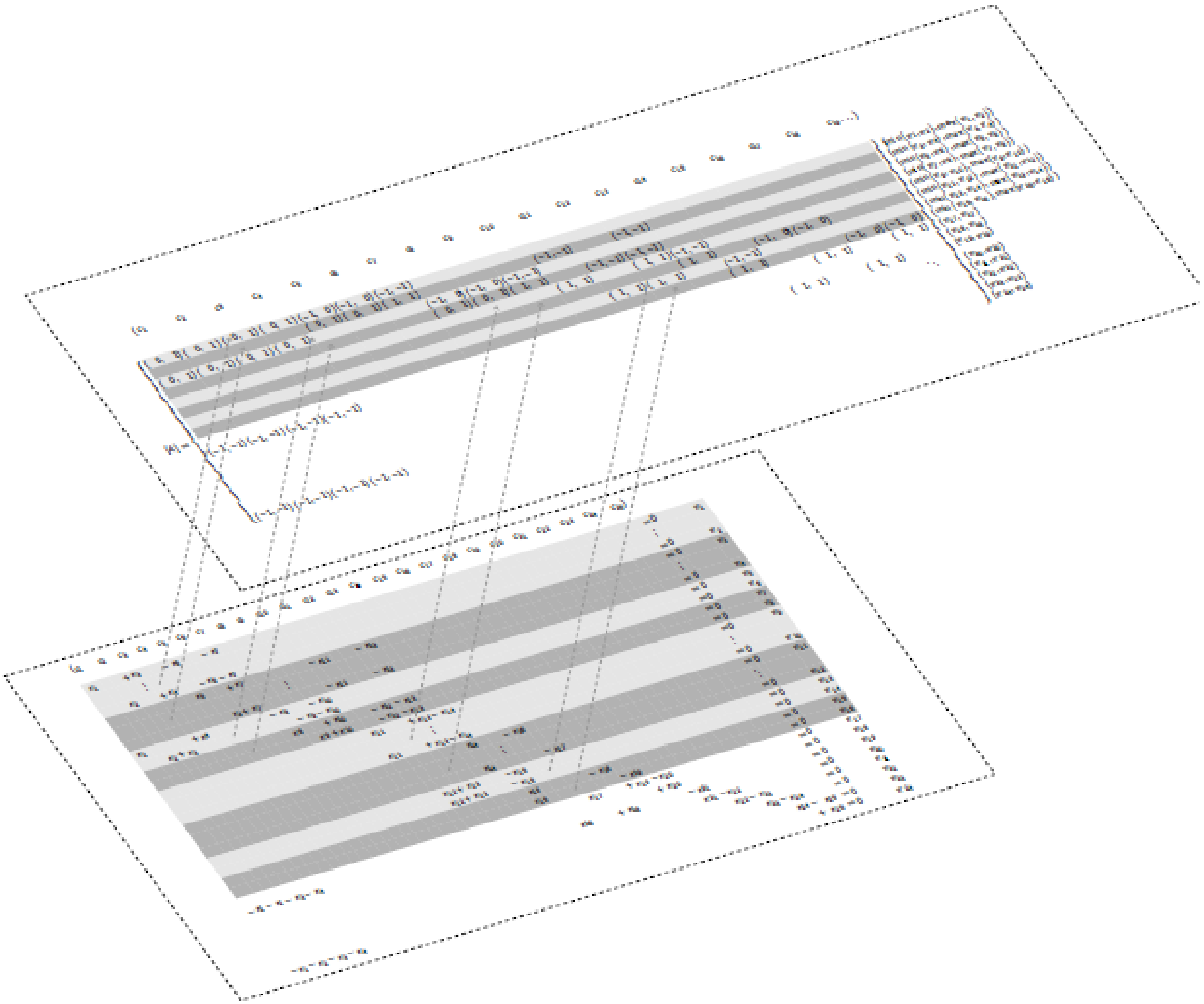
$$[\overline{c}_j] \coloneqq [c_j] - [\pi]^T \cdot [a_j] \leq c_j - \pi^T \cdot a_j = \bar{c}_j.$$

- Use the coarse reduced cost for pricing in the fine model.

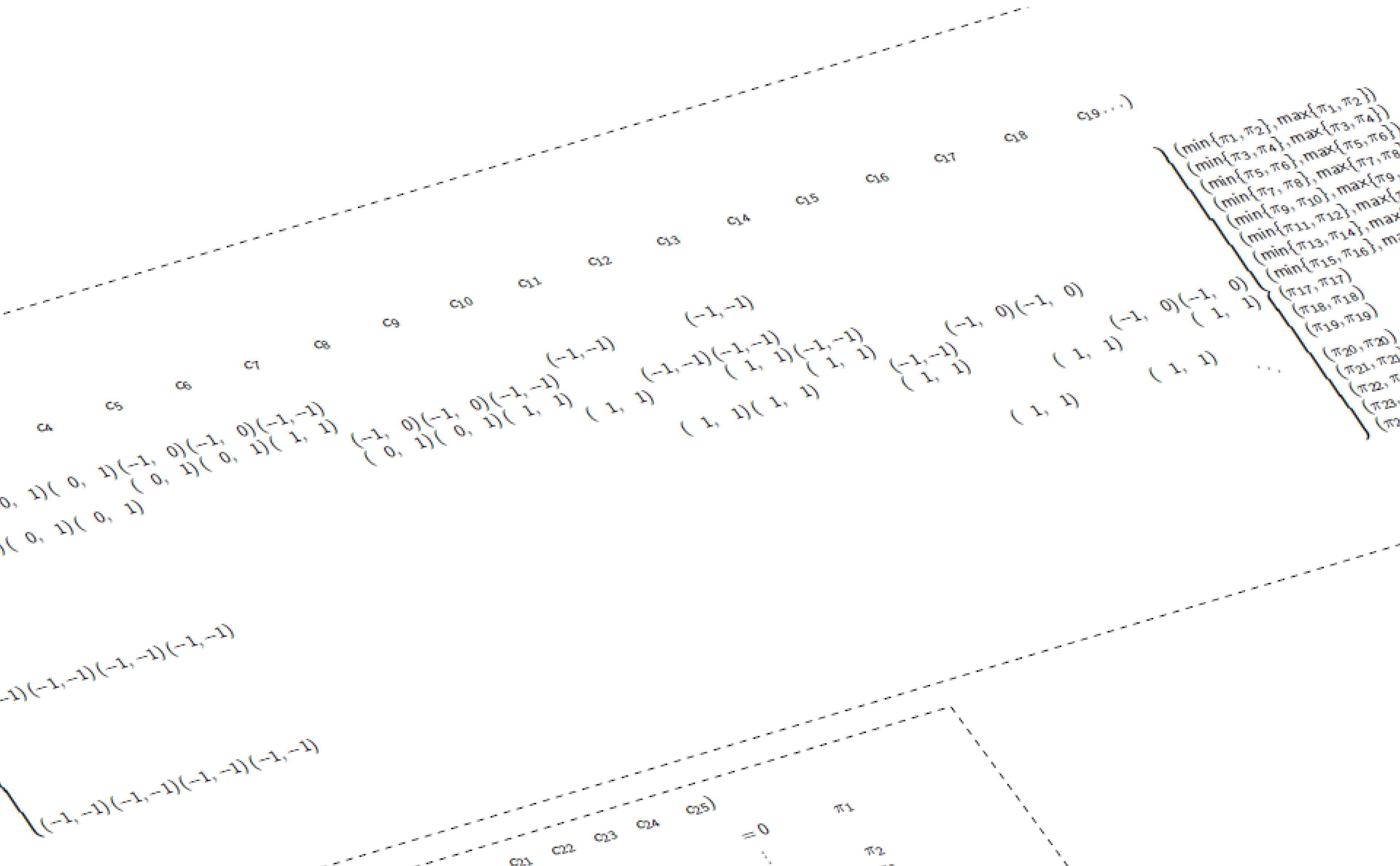
Coarse-to-Fine Method: Example

$$
\begin{array}{lll}
(c_1 \; c_2 \; c_3 \; c_4 \; c_5 \; c_6 \; c_7 \; c_8 \; c_9 \; c_{10} \; c_{11} \; c_{12} \; c_{13} \; c_{14} \; c_{15} \; c_{16} \; c_{17} \; c_{18} \; c_{19} \; c_{20} \; c_{21} \; c_{22} \; c_{23} \; c_{24} \; c_{25}) & & \\
x_1 + x_3 - x_5 - x_7 & = 0 & \pi_1 \\
\vdots & \vdots & \\
x_2 + x_4 - x_6 - x_7 & = 0 & \pi_2 \\
x_5 + x_7 - x_{11} - x_{13} & = 0 & \pi_3 \\
\vdots & \vdots & \\
x_6 + x_7 - x_{11} - x_{13} & = 0 & \pi_4 \\
x_1 + x_4 - x_8 - x_{10} & = 0 & \pi_5 \\
x_2 + x_3 - x_9 - x_{10} & = 0 & \pi_6 \\
x_8 + x_{10} - x_{12} - x_{13} & = 0 & \pi_7 \\
x_9 + x_{10} - x_{12} - x_{13} & = 0 & \pi_8 \\
x_{11} + x_{13} - x_{14} & = 0 & \pi_9 \\
\vdots & \vdots & \\
x_{11} + x_{13} - x_{14} & = 0 & \pi_{10} \\
x_{14} - x_{16} & = 0 & \pi_{11} \\
\vdots & \vdots & \\
x_{14} - x_{17} & = 0 & \pi_{12} \\
x_{12} + x_{13} - x_{15} & = 0 & \pi_{13} \\
x_{12} + x_{13} - x_{15} & = 0 & \pi_{14} \\
x_{15} - x_{18} & = 0 & \pi_{15} \\
x_{15} - x_{19} & = 0 & \pi_{16} \\
x_{17} + x_{19} - x_{20} & = 0 & \pi_{17} \\
-x_1 - x_2 - x_3 - x_4 + x_{20} & = 0 & \pi_{18} \\
x_{16} + x_{18} - x_{21} & = 0 & \pi_{19} \\
x_{21} - x_{22} & = 0 & \pi_{20} \\
x_{22} - x_{23} & = 0 & \pi_{21} \\
x_{23} - x_{24} & = 0 & \pi_{22} \\
x_{24} - x_{25} & = 0 & \pi_{23}
\end{array}
$$

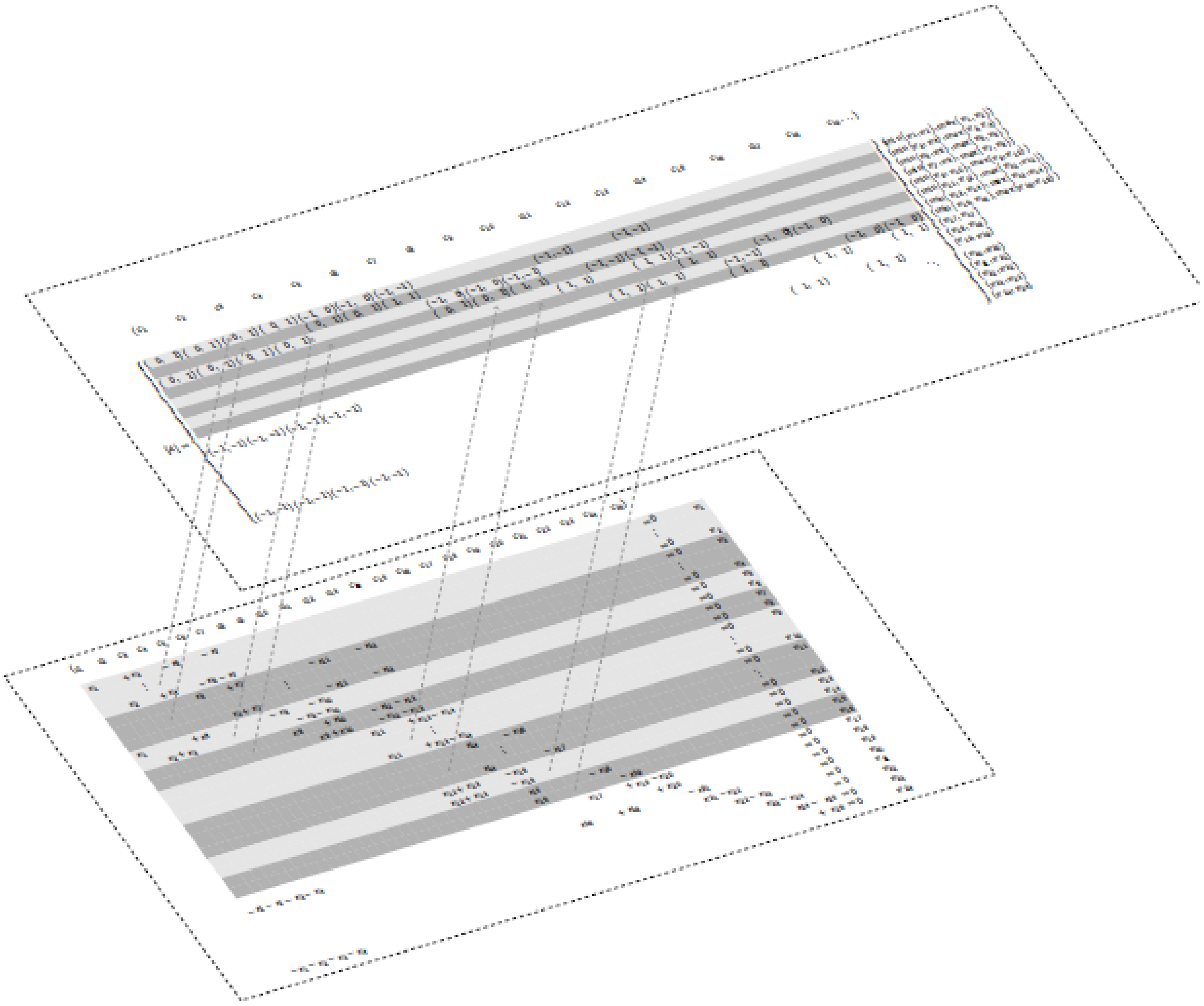
Coarse-to-Fine Method: Example



Coarse-to-Fine Method: Example



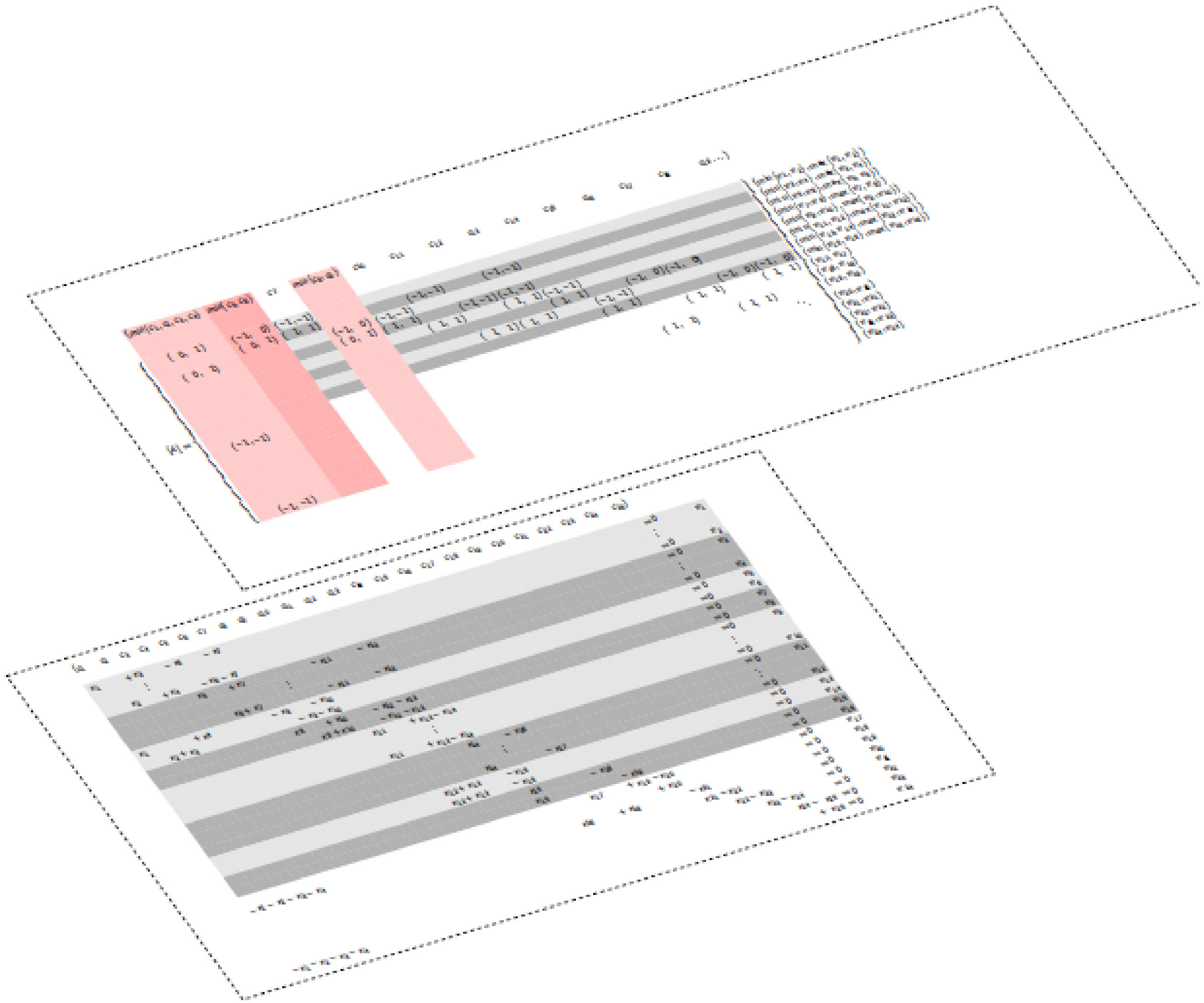
Coarse-to-Fine Method: Example



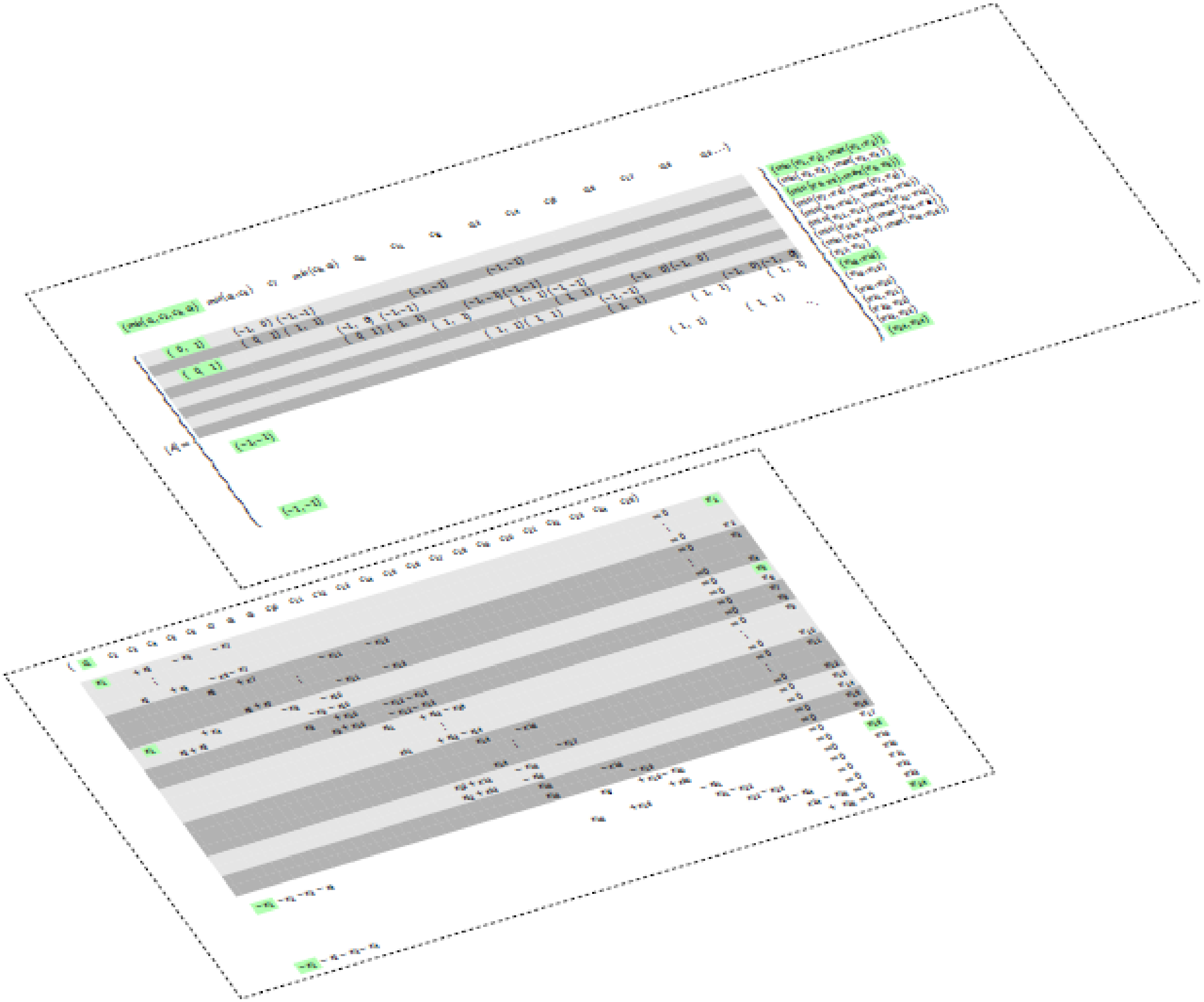
Coarse-to-Fine Method: Example

MODAL

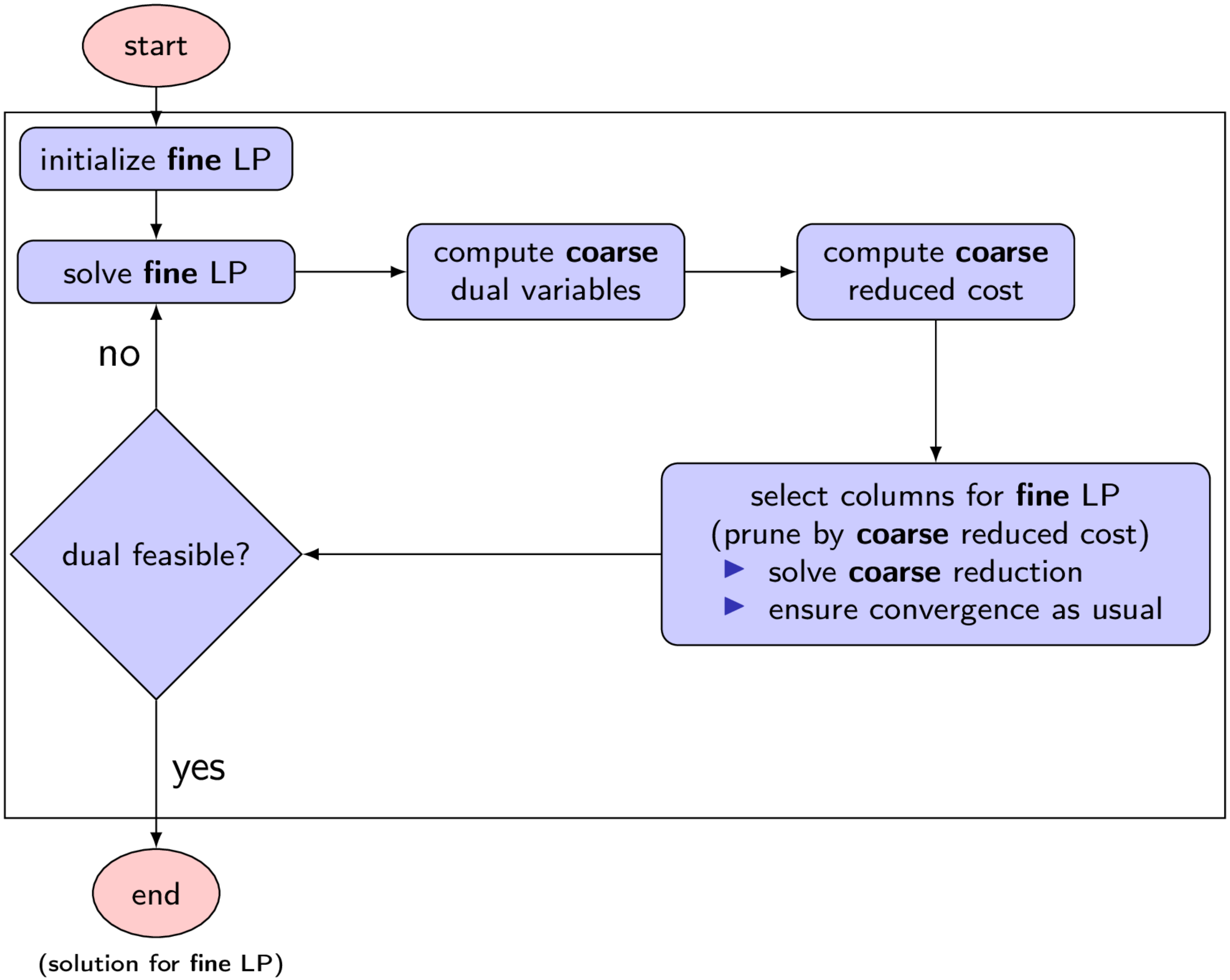
Coarse-to-Fine Method: Example



Coarse-to-Fine Method: Example



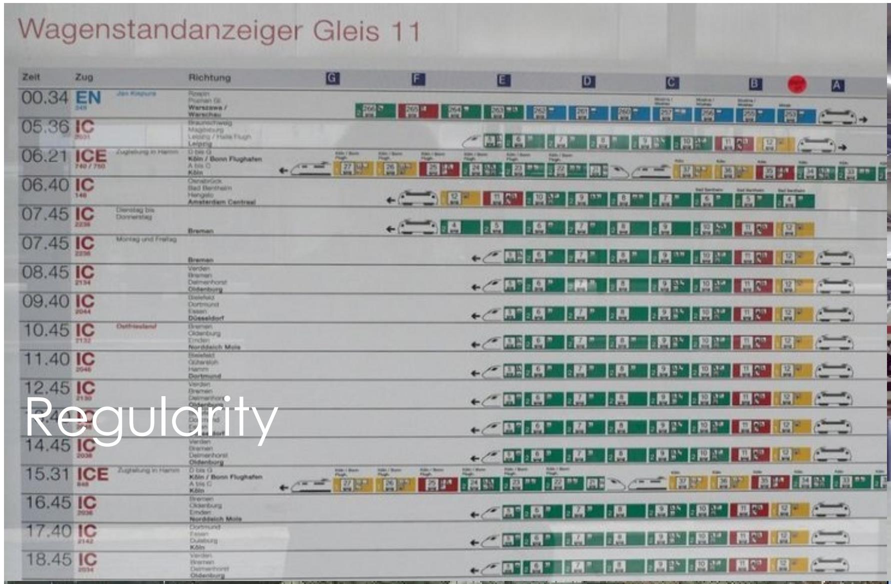
Coarse-to-Fine Method: Coarse Pricing



Coarse-to-Fine Method: Performance Gain

- The solution of the root LP is 1.5-2x slower
- The solution of the node LPs is 10-20 x faster
- Memory consumption is much smaller

Railway Constraints



Maintenance: Service Intervals

Blue: timetabled trips

Green: 4000 km treatment

Dark gray: 8250 km treatment

Yellow: 33000 km treatment

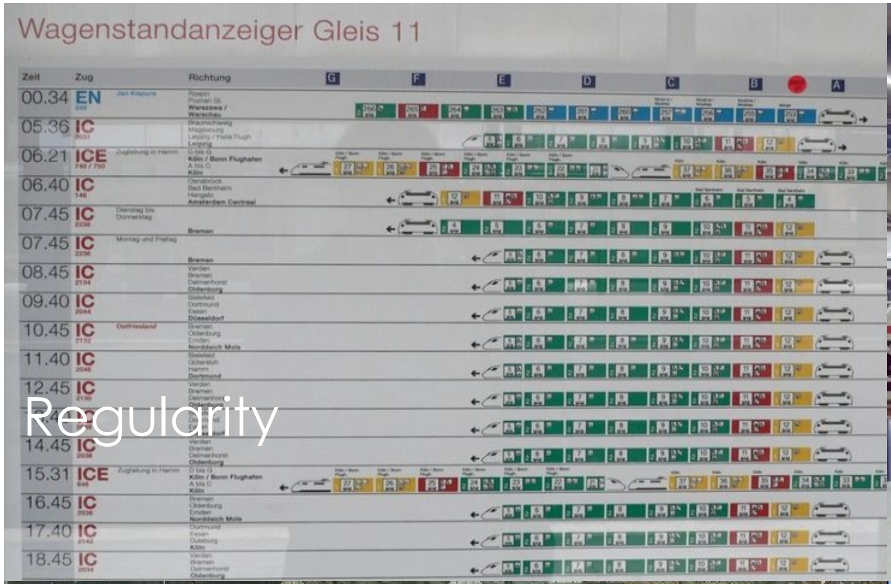
Pink: 66000 km treatment

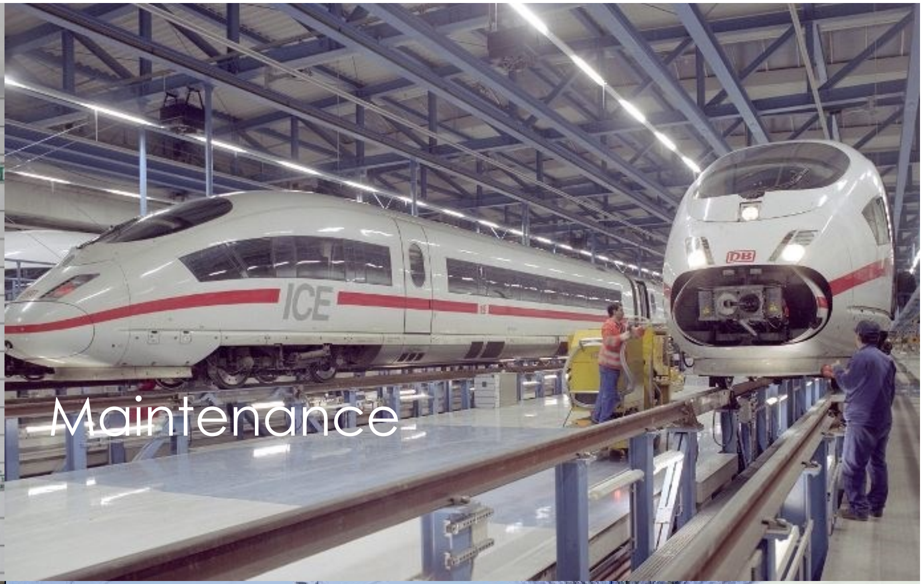
Red: 198000 km treatment

Light gray: 15 days treatment

Turquoise: 30 days treatment

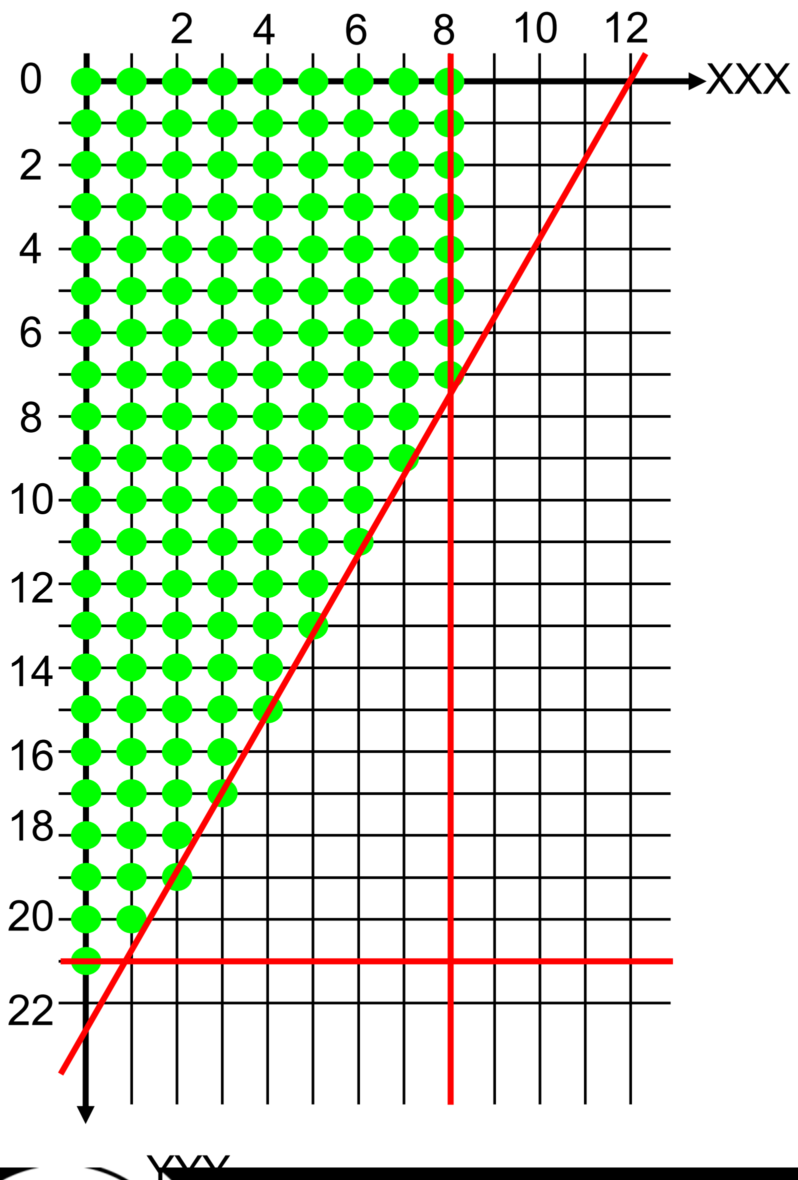
Railway Constraints





Photos courtesy of DB Mobility Logistics AG

Parking: Keeping It Simple



Siding	Length/m	Feasible Assignments
1	570	XXX, YYY, XXX+YYY, YYY+YYY
2	480	XXX, YYY, YYY+YYY
3	430	XXX, YYY, YYY+YYY
4,5,6	420	XXX, YYY, YYY+YYY
7	410	XXX, YYY, YYY+YYY
8	390	XXX, YYY
9,10,11	240	YYY
12,13,14	210	YYY

Vehicle Rotation Planning Model

$$\min \sum_{a \in H} c_a x_a , \qquad \text{(objective)}$$

$$\sum_{a \in H(t)} x_a = 1 \qquad \forall t \in T, \qquad (1)$$

$$\sum_{a \in H(v)^{\text{in}}} x_a = \sum_{a \in H(v)^{\text{out}}} x_a \qquad \forall v \in V, \qquad (2)$$

$$w_a^l \leq \sum_{a \in H(a)} U_l \, x_a \qquad \forall a \in A, \, l \in L, \qquad (3)$$

$$\sum_{a \in A(v)^{\text{out}}} w_a^l - \sum_{a \in A(v)^{\text{in}}} w_a^l = \sum_{a \in H(v)^{\text{out}}} r_l^v(a) \, x_a \qquad \forall v \in V, \, l \in L, \qquad (4)$$

$$\sum_{a \in H} r_b(a) \, x_a \leq U_b \qquad \forall b \in B, \qquad (5)$$

$$x_a \in \{0, 1\} \qquad \forall a \in H, \qquad (6)$$

$$w_a^l \in [0, U_l] \subset \mathbb{Q}_+ \qquad \forall a \in A, \, l \in L \qquad (7)$$

Real World Example: Scenario 1

Input	#	Objective	Goal
Timetabled trips	798	Coverage	100%
Connections	171	Rows	Minimum
Maintenance interval • Small: every 12500 km @ 1 depot • Monthly: every 25000 km @ 1 depot • Big: every 50000 km @ 1 depot	3	No of. maintenance services	Minimum
Stations	14		
Depots	7		

Objective	Reference solution	VS-OPT rail
Rows	20 + 300 km deadhead	19 + 300 km deadhead
CPU time (hh:mm)	—:—	00:20

Real World Example: Scenario 2

Input	#	Objective	Goal
Timetabled trips	1292	Trip coverage	100%
Connections	1009	Rows	Minimum
Maintenance intervals • Refuel: every 600 km @ 10 depots • Small: every 15000 km @ 1 depot • Big: every 60000 km @ 1 depot)	3	No of maintenance services	Minimum
Stations	26		
Depots	34		

Objective	Reference solution	VS-OPT rail
Rows	29 + 5500 km deadhead	26 + 3300 km deadhead
CPU time (hh:mm)	—:—	08:48

Delay Resistant Train Rotations

Measuring Robustness as Expected Propagated Delay

Initially set arrival delay $\mathrm{AD}_t = 0$.

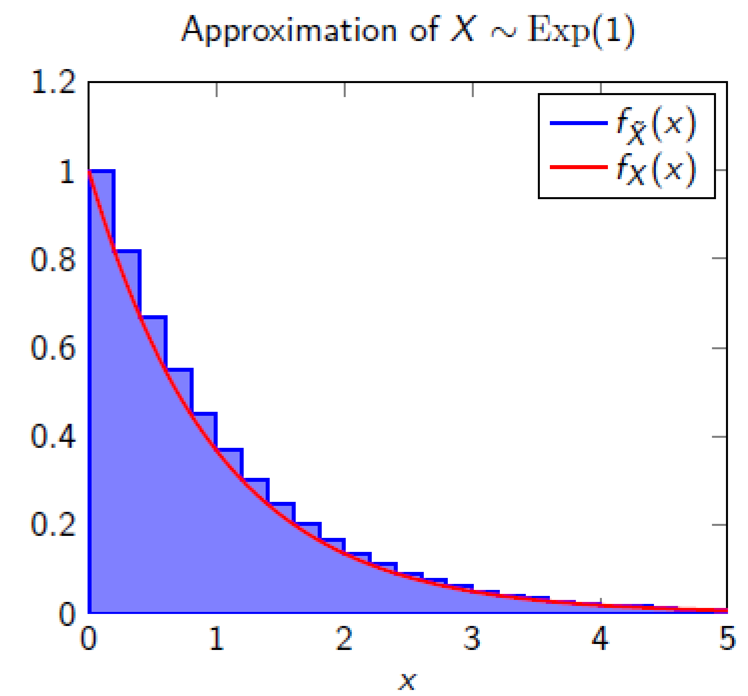
$$\mathrm{AD}_t = \underbrace{\max_{s \in C_t}\Big(\overbrace{\max(\mathrm{AD}_s + c - b_{s,t}, 0)}^{\text{connection delay (CD)}}\Big)}_{\text{departure delay (DD)}} + \overbrace{\mathbf{D_t}}^{\text{primary delay}}$$

For a trip t:

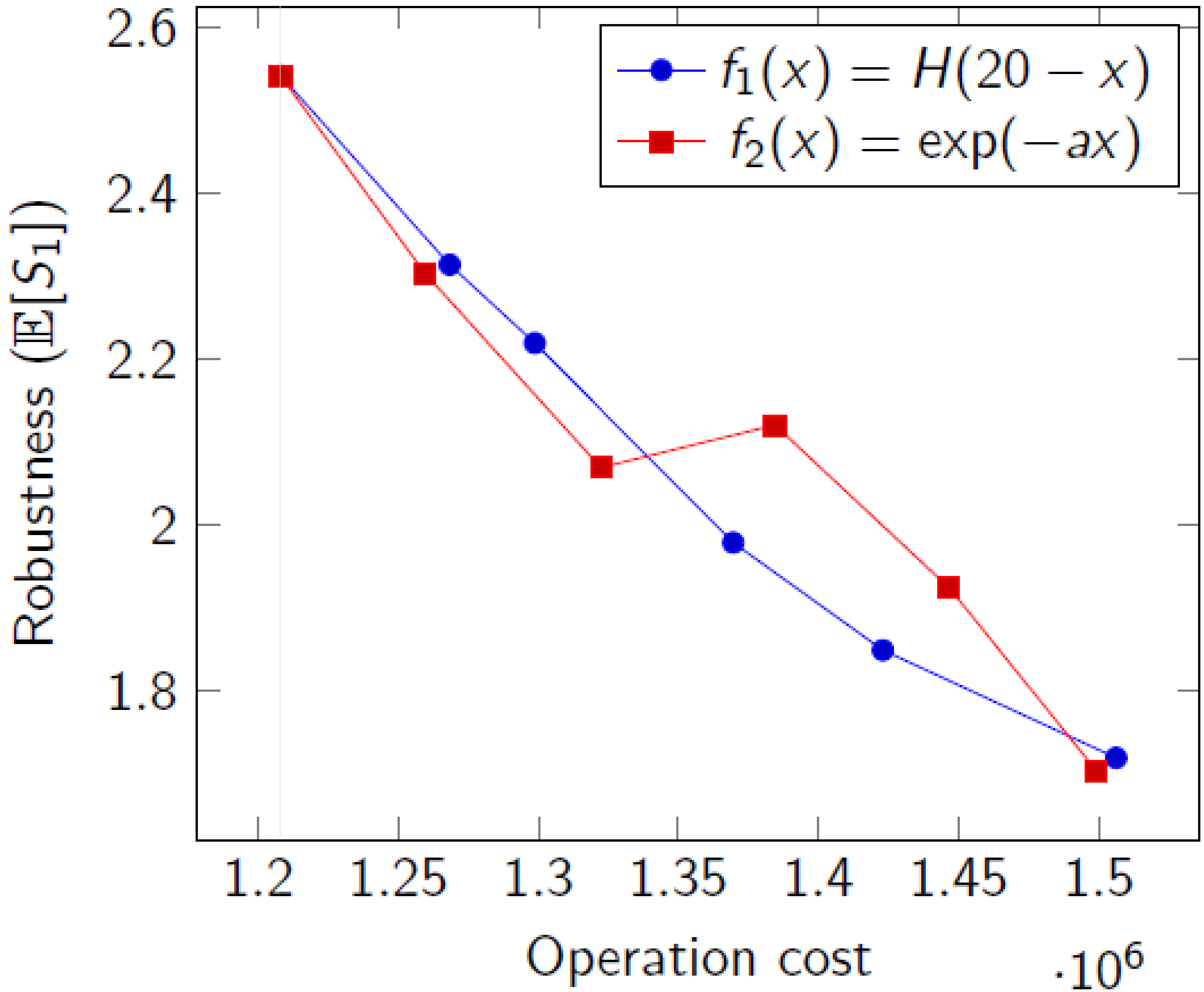
- C_t - set of connecting trips
- $b_{s,t}$ - stop over time
- D_t - primary delay (e.g. breakdowns, disruptions)
- c - turn over time (e.g. cleaning, crew changes)

Minimizing the Expected Propagated Delay

- Computing the propagated delay distribution requires a convolution
- Approximate primary delay using discrete random variables
- Numerical effort is quadratic in the number of discretization intervals
- Approach: penalize small buffers, verify EPD



EPD: -9% EPD for +5% Cost?



The Price of Regularity: Case Study

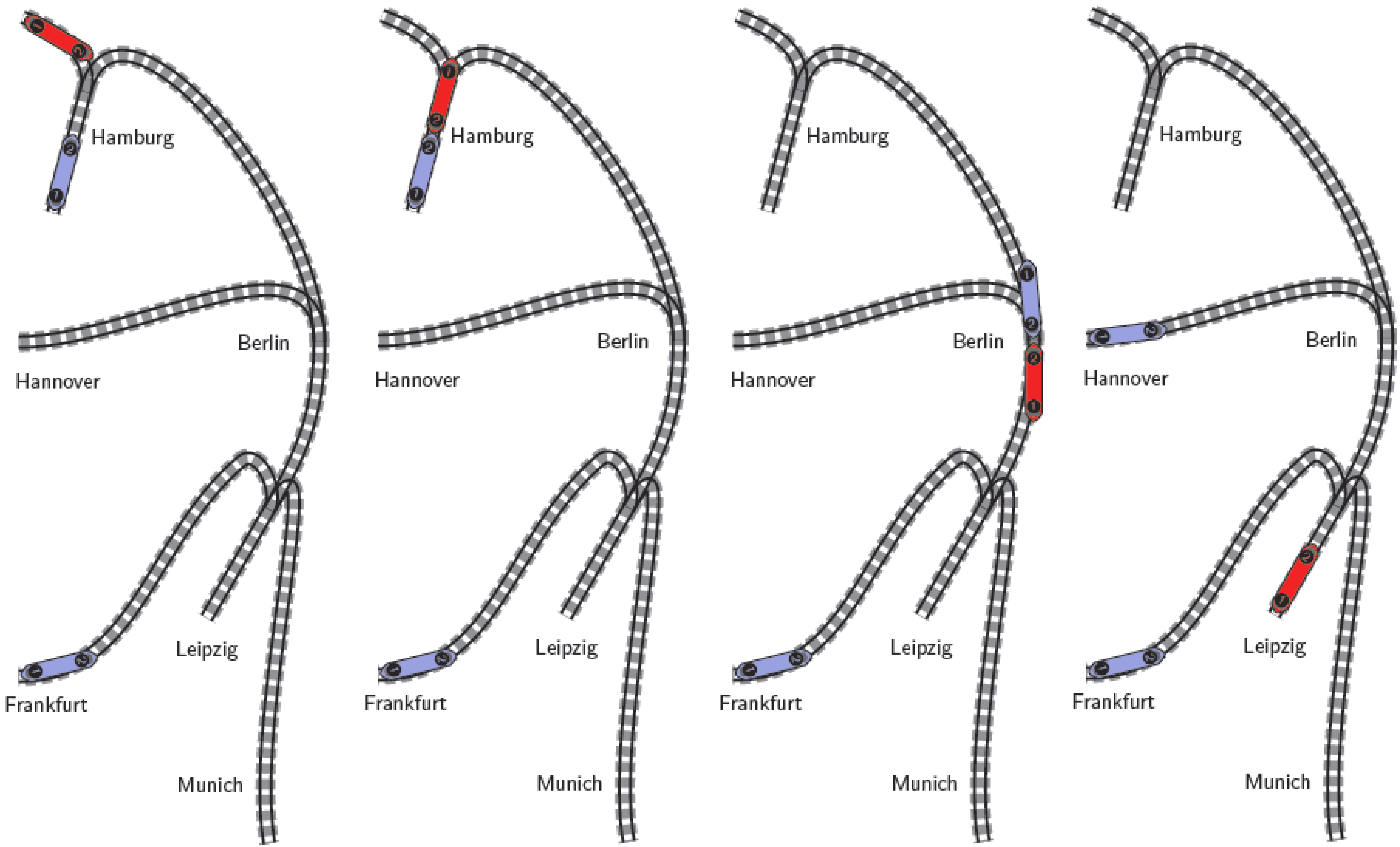
Real world scenario

- 670 timetabled trips (127 trains)
- 52 locations
- vehicle compositions of size at most two
- 4 946 356 hyperarcs

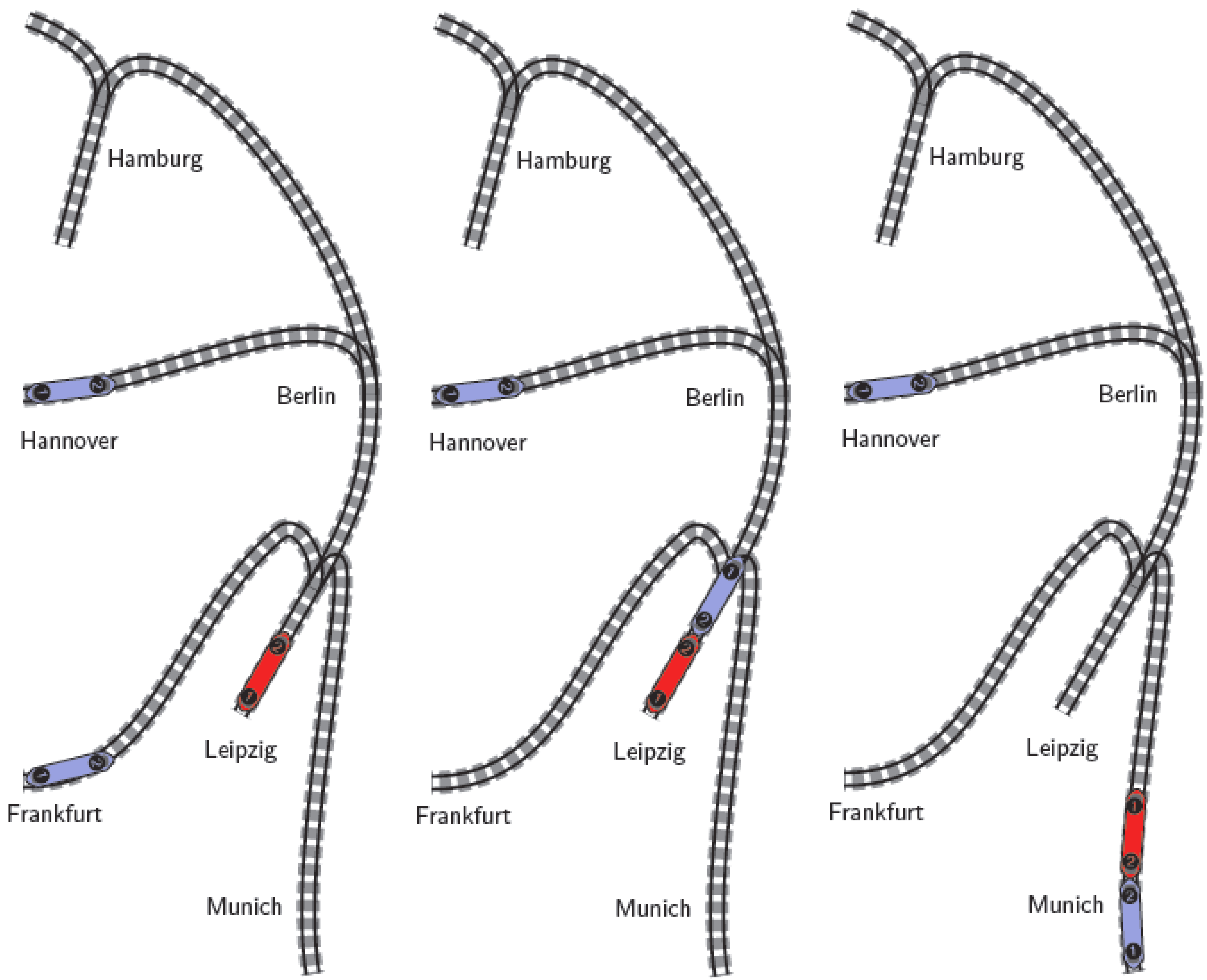
Bi-criteria objective function

- Minimize operational cost including
  - cost for rolling stock
  - cost for deadhead trips
  - cost for additional turn around trips
  - cost for violating planned turn times
- Maximize regularity (i.e., minimize irregularity penalty R)
- Weighted sum method

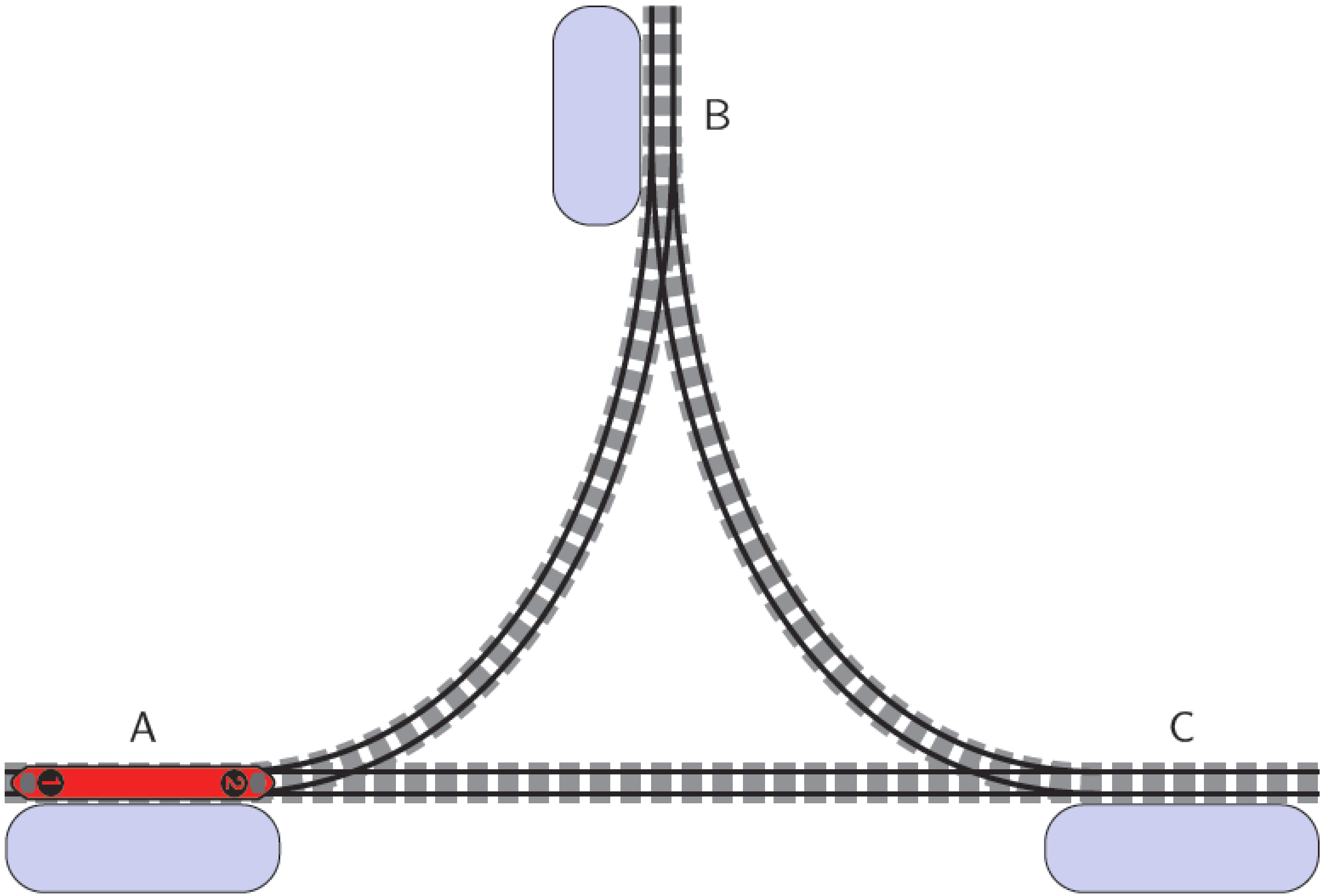
PoR: Complex Coupling Requirements



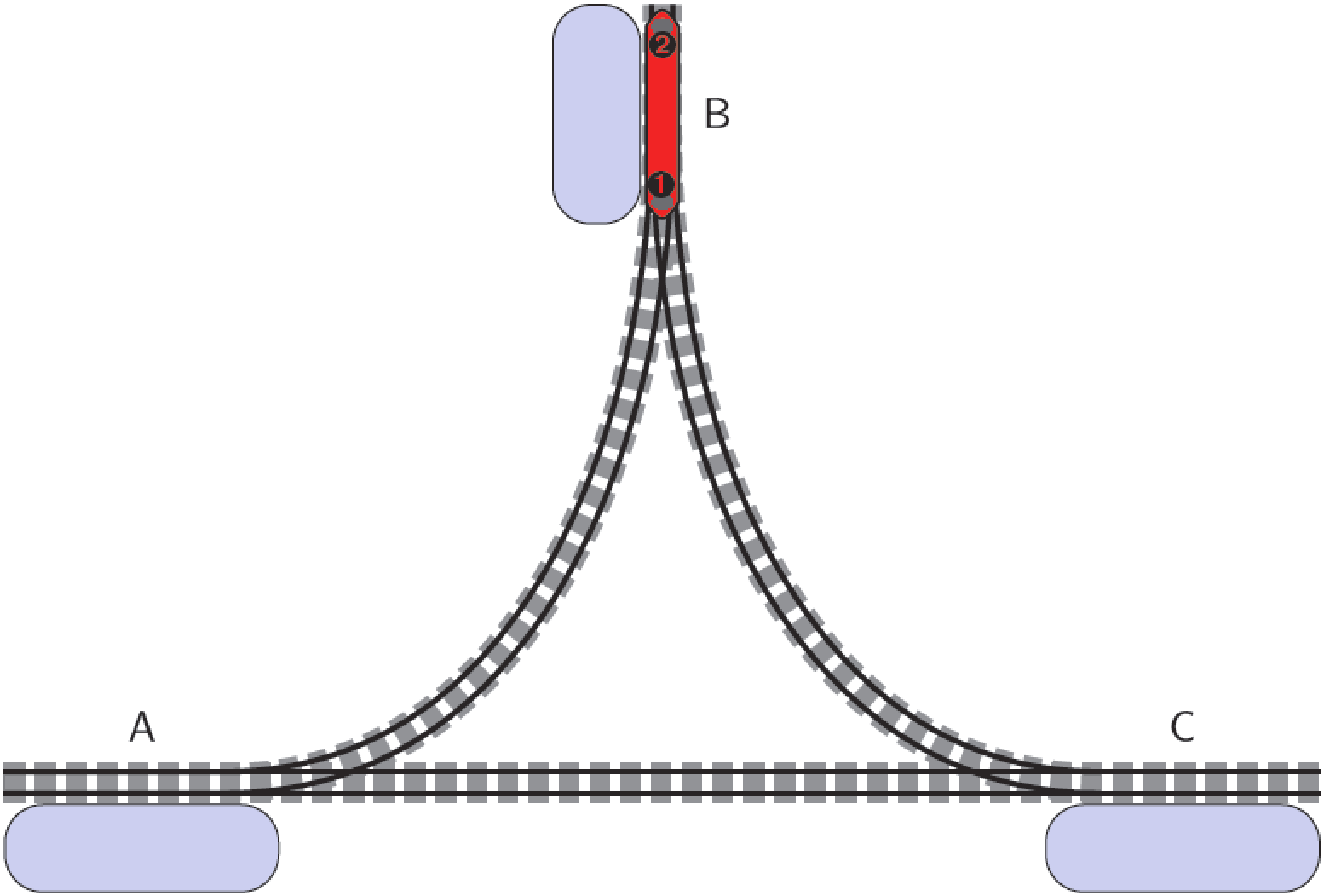
PoR: Complex Coupling Requirements (cont'd)



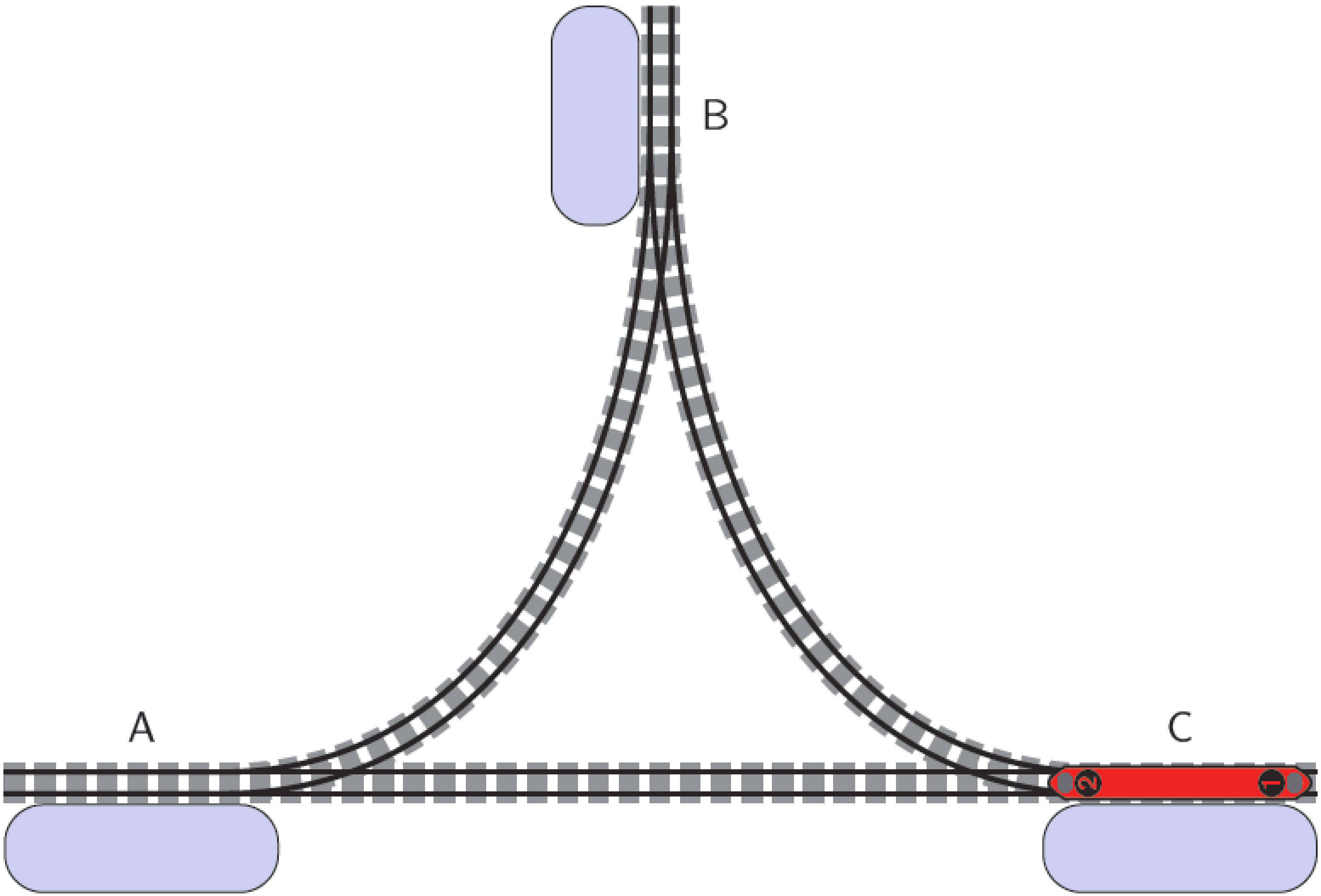
PoR: Additional Turn-around Trips



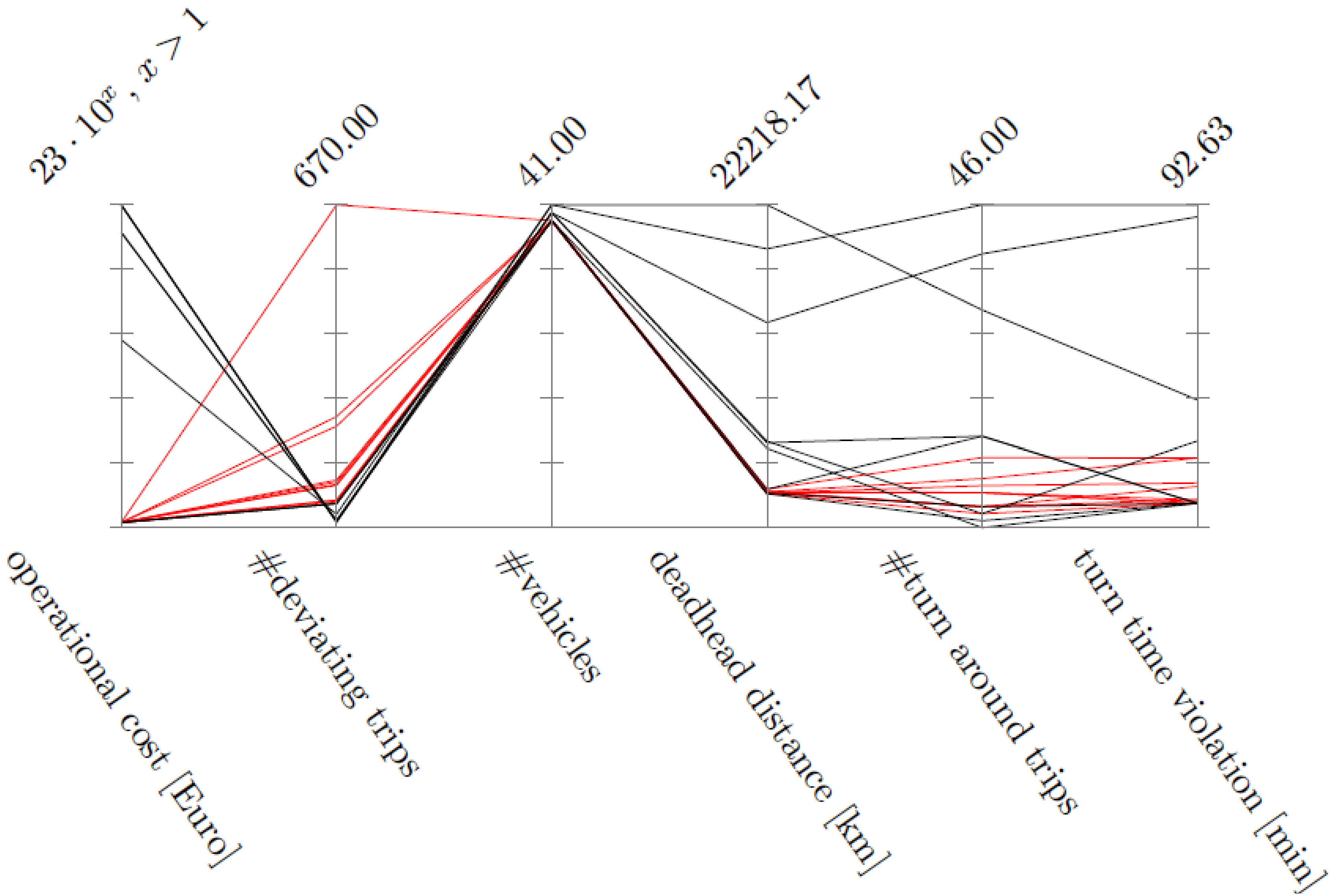
PoR: Additional Turn-around Trips



PoR: Additional Turn-around Trips



PoR: Results



$23 \cdot 10^x, x > 1$
670.00
41.00
22218.17
46.00
92.63
operational cost [Euro]
#deviating trips
#vehicles
deadhead distance [km]
#turn around trips
turn time violation [min]

Thank you for your attention

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