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# Vehicle Rotation Planning for ICE High Speed Trains

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joint work with O. Heismann, M. Reuther, T. Schlechte, S. Weider et. al.

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## InterCity Express (ICE) High Speed Train







#### ... must be used efficiently



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#### Rotor 1.0

- ▶ in production since 7 / 2013
- integrates all technical details

#### Rotor 2.0

- ▶ in production since 3 / 2014
- reduced memory consumption
- implements re-optimization





#### Research Campus MODAL @ ZIB







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#### **MODAL:** Industry Partners



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MODAL: Mathematical Optimization & Data Analysis Laboratories



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#### MODAL Rail-Lab: Phases

- Builds on earlier work with DB (VR-OPT)
- ▶ 3 phases of 5 years, phase I 2015-2019
- ► 4 positions, 2 industry + 2 BMBF



#### Fully Dated Vehicle rotation planning

Mathematical Models and Algorithms

Integer Programming Integrated Flow and Path Model Bundle Method <u>Coarse-to-Fine Method</u>

Combinatorial Optimization Hyperassignments Hyperflows Tree Decompositions Data Analysis and System Integration

System Integration Data acquisition Interfaces Optimization cores

Data Analysis Railway Requirements Case studies and calibration Visualization and statistics



#### **ICE Network: Connections**



## Timetabled Trips: 1 Day



#### Timetabled Trips: Standard Week



#### Vehicle Rotation: 1 Week



#### Graphics: JavaView, MATHEON F4



#### Vehicle Rotation: 5 Weeks



#### Graphics: JavaView, MATHEON F4

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#### Rotation Plan: Follow-on Trip Assignment

(Blue: Timetabled Trips, Red: Deadhead Trips)



#### Graphics: JavaView, MATHEON F4



#### Again: Follow-on Trip Assignment



#### Railway Constraints

Wagenstandanzeiger Gleis 11

Zeit	Zug		Richtung	G	F		D	C	B		IA
00.34	EN	Jan Konpura	Rosers Poenan GL Warstowe /			sa - 500 - 5 (500 -				ing .	-
05.36	IC		BraunicTreng Magneturp Lengrg / Hulls Flugh	1	And and a second second						( <u></u> )+
06.21	ICE	Zugleburg in Harris	Criping Dible G Köln / Bonn Flughaten A bis C		Tage - Ser -	HELE DANS					) ->
06.40	IC		Köln Osnabrück Slad Ekrothalm Hengelo	C					and the sectors	Ref Instance	
07.45	IC	Dierstag bis Dorrienting	Amaterdam Centrael								
07.45	IC	Montag und Freitag	Bremen								
08.45	IC .		Bremen Verden Bremen Damerkost	C' I ILES						1270	
09.40	IC		Divertision Divertisional Examination								
10.45	IC	Optimized	Dramon Oldantsung Einden			+					
11.40	IC 2046		Biolefield Gotavelun Hanzn			+				12	-
12.45	IC		Verden Verden Bremen Demenhon			+					
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14.45	IC		Verden Diemen Demenhonst Offensione			+ @ 27 22	M . M	· PA 12" - 14			-
15.31	ICE	Zugtaliung in Harnin	O bis G Köin / Bonn Flughaten A bis C Köin	+							
16.45	IC		Bronsen Chierdsung Einden Reardslatisch Male			+ - 1 1	- M - M			13° (	
17.40	IC		Dorbreand Extern Dutations Köln			+ @				1	
18.45	IC		Verden Ekonen Ostreetkont			+ C II II				(E)= 6	



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Aaintenance

Train Composition

## Timetable Regularity $\rightarrow$ Rotation Regularity

Wagenst	andanzeig	er Gleis	11						130
Zeit Zug	Richtung	G	E	E	D	C	E	-	
00.34 EN	Roman Roman GL Poemen GL Warszawa /	2000	- 200 a 20						
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06.21 ICE Jugini	Leipzig arg in Hamm D bas 0 Köln / Bonn Flughaten A bis C		And the And the And	Hand Dave Hand	Aller Case		100 100 100	( <u></u> )→	
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11.40 10	Gütersloh Harren Dortmund			+ @ 18 1	- <b>A</b>   A	·	2ª 📕 🖓 😫		
12.45	Bremen Detmanhorst Oldenburg			+ C	* <b>M</b> * <b>H</b>	· P. * . P.	* M: 8		
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20.45	Dontmand Vierdan Enemen Dateschorst (Distantions)	1		+ CH	· A. H		- 8.		

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#### Modeling Rotation Regularity ...



#### Modeling Rotation Regularity via Hyperarcs



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## Hyperassignment Solution





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#### Bipartite Hypergraph Model





A hypergraph G is called bipartite if

- its vertex set can be written as the disjoint union of two vertex sets U and V with the same size |U| = |V|, and
- every hyperedge  $e \in E$  has the same number  $|e \cap U| = |e \cap V|$  of vertices in U and V.

We then represent G as a triple G = (U, V, E).



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A hyperassignment is a subset H of E such that there is exactly one incident hyperedge for every vertex.



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## The Hyperassignment Problem

#### **Definition (Hyperassignment Problem)**

Input: A bipartite hypergraph G = (U, V, E) with edge costs  $c_e \in \mathbb{R}$ . Output: A minimum cost hyperassignment  $H^*$  in G, i.e., a hyper-assignment  $H^*$  s.t.

 $c(H^*) = \min\{c(H), H \text{ is a hyperassignment in } G\}$ 

or the statement that no hyperassignment exists.

$$\min \begin{array}{c} c^T x \\ x(\delta^+(v)) = 1 \quad \forall v \in U \cup V \\ x(\delta^-(v)) = 1 \quad \forall v \in U \cup V \\ x \in \{0,1\}^E \end{array}$$

The HAP is a special type of set partitioning problem.

## Complexity Results

#### Theorem (B., Heismann [2011], Heismann [2014])

- 1. The HAP is NP-hard and APX-hard, even for bipartite hypergraphs with maximum hyperedge size 4.
- 2. The set packing/covering relaxations of the HAP are NPhard, even for bipartite hypergraphs with maximum hyperedge size 6.
- 3. The LP/IP gap can be arbitrarily large.
- 4. The determinants of basis matrices can be arbitrarily large.





#### Solution of the LP Relaxation

Fractional solution, cost = 0.615.



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### Solution of the LP Relaxation

- Fractional solution, cost = 0.615.
- ► The red hyperedge clique inequality separates this solution.
- Cliques can be separated efficiently by exploiting a "partitioning structure".





#### Partitioned Hypergraphs



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#### Theorem (B., Heismann [2012])

Every HAP in a bipartite hypergraph G = (U, V, E) can be polynomially transformed into a HAP in a partitioned hypergraph with  $d = 0.5 \max_{e \in E} |e|$ .



#### Theorem (B., Heismann [2011])

Every (hyperedge) clique in a partitioned hypergraph is a subset of the incident hyperedges δ(P) of some part P.
The (hyperedge) conflict graph contains no holes of any size and no antiholes of size < 7.</li>



#### Solution of the LP Relaxation

- Fractional solution, cost = 0.635.
- Consider the 7=2·3+1 cliques associated with the vertices  $v_1, v_3, v_4, u_2, u_3, u_4$  and the clique  $\{v_5, v_6, u_3, u_4\}, \{v_5, u_3\}, \{v_5, u_4\}$ .





#### Solution of the LP Relaxation

- Fractional solution, cost = 0.635.
- Consider the 7=2·3+1 cliques associated with the vertices  $v_1, v_3, v_4, u_2, u_3, u_4$  and the clique  $\{v_5, v_6, u_3, u_4\}, \{v_5, u_3\}, \{v_5, u_4\}$ .
- Every red hyperedge is contained in at least two of these cliques.
- We can take at most three of these edges.





## Odd Set leqs for the (Perfect) Matching Problem



$$\sum_{e \in E} \left[ \frac{||v| + |v| +$$

 Complete description of the matching polytope (together with the degree and non-negativity constraints), Edmonds [1965]

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### Odd Clique Set leqs for General Hypergraphs



$$\sum_{e \in E} \left\lfloor \frac{|\{v \in V' : e \in \delta(v)\}|}{p} \right\rfloor x_e \le \left\lfloor \frac{|V'|}{p} \right\rfloor \quad \forall V' \subseteq V$$

Related to clique set inequalities by Pêcher & Wagler [2006]

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## Odd Clique Set leqs for General Hypergraphs



#### Theorem (B., Heismann [2011])

Let Q be a set of at least three hyperedge cliques in G = (V, E),  $2 \le p \le |Q|$  be an integer number,  $r := |Q| \mod p$ , and  $q_e := |\{Q \in Q : Q \ni e\}|$ . Then

$$\sum_{e \in E} \left( \left\lfloor \frac{q_e}{p} \right\rfloor + \max\left\{ 0, \frac{q_e \mod p - r}{p - r} \right\} \right) x_e \le \left\lfloor \frac{|\mathcal{Q}|}{p} \right\rfloor.$$
#### Hyperassignment Solution

▶ Integer solution, cost = 1.010.



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Train Composition



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## Train Composition

Zeit	Zug		Richtung	Α		В		С		D		E
06.12	<b>D</b> 773	Di bis Sa 29 Mai 2005 bis 11 Jun 2006 auch 29 Mai 2006	Neumünster Hamburg Hamburg Stuttgart		14	12	11	10	<u> </u>			
06.12	<b>AZ</b> 773	außer So 12.Jun 2006 bis 9.Dez 2006	Neumünster Hamburg Hamburg	<u>~</u> ==	14	12	11	10	2 9 4 0			
06.12	ALX 773	11.Dez 2005 bis 27 Mai 2006	Neumünster Hamburg Hamburg		14	13	12	11 1	10			
07.12	ICE		Neumünster Hamburg Hamburg	/ 28 L	27	26 1	24	23 🛄	22 🛦 📖	31		
08.12	<b>EC</b>		München Neumünster Hamburg Hamburg		14	12	11	10	9 <u>4</u> 1			
12.38	UEx	50 11 Daz 2005 bis 18 Daz 2005 50 Dag 2005 bis 18 Daz 2005	Stuttgart Neumünster Hamburg Hamburg		14	12	11	10	2 <b></b>			
12.38	<b>X</b> 927	auch 26 Dez 2006 No und So 17 Apr 2006 bis 23 Apr 2006	München Neumünster Hamburg Hamburg		14	12 4	11	10	9			
12.38	RE	5a 1 Mai 2006 bis 21 Mai 2005 auch 26 Dez 2005, 1 Mai 2006	München Neumünster Hamburg Hamburg	<u></u>	14 4 5		11	10	2 🛶 🗖	7		
12.38	THA	12 Dar 2005 bis 25 Dar 2005 nicht 18 Dar 2005 aufär 50	München Neumünster Hamburg Hamburg			12		10	2 🛶 🔺	7	6 5	5
12.38	MET	26.Dez 2005 bis 9.Apr 2006 nicht 18.Dez 2005, 26.Dez 2005 Di bis 58 10.Apr 2005 bis 7.Mei 2005	Nürnberg Neumünster Hamburg			-			2	2 🚊 2	<u> </u>	2 2
12.38	<sup>927</sup> CIS	auch 10 Apr 2006, 16 Apr 2006, 24 Apr 2006 auch 30 Apr 2006 aufer So	Nürnberg Neumünster Hamburg							-		
16.12	927 NZ	B Mai 2006 bis 27. Mai 2006	Hamburg Nümberg Neumünster Hamburg	<u></u>	14	12	11	10	2 🚆 👗			
18.38	675 EN	28 Mai 2006 bis 9 Daz 2006	Hamburg Stuttgart Neumünster Hamburg		14 📼	12	11	10	<u> </u>	1		
18.38		11.Dez 2005 bis 27 Mai 2006	Hamburg Basel SBB Neumünster	<u></u>	2 🗖	2 2	2 3	2 4	2 1	2 6 2 2	7	2 🚔 🚺
.0.00	809		Hamburg Basel SBB		14	12	11	10	2 🛁 👗 🕻	2 7 2	6	2 5 2
				A		В		С		D		E

# Rare Train Composition Example



#### Train Composition: Type, Order, Orientation



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#### Hypergraph Model: Possible Train Compositions



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## Hypergraph Model: Arrival and Departure Nodes



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#### Hypergraph Model: Single Traction



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#### Hypergraph Model: Double Traction



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## Hypergraph Model: Triple Traction



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## Hypergraph Model: Pass-Through Connections





## Hypergraph Model: Pass-Through Connections





#### Hypergraph Model: All Connections





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#### Hypergraph Model: Zoom



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#### Hypergraph Model: Timetabled Trips



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#### Hypergraph Model: Connections



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#### Hypergraph Model: All Connections



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#### Hypergraph Model: Two Daily Trains





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## Hypergraph Model: Max Regularity (0 Idle Days)





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## Hypergraph Model: Max Regularity (1 Idle Day)





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## Hypergraph Model: Max Regularity (2 Idle Days)







## Hypergraph Model: Max Regularity (3 Idle Days)





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## Hypergraph Model: Max Regularity (4 Idle Days)





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## Hypergraph Model: Max Regularity (5 Idle Days)







## Hypergraph Model: Max Regularity (6 Idle Days)





## Hypergraph Model: Max Regularity (All Options)





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## Hyperflow Model

Vehicle Rotation Planning Problem

Cover all timetabled trips by rotations such that turns and train composition are regular.

Hypergraph Multi Commodity Flow Problem

Find a cost minimal hyperflow such that every node configuration is covered by exactly one hyperarc.

Hyperarcs for

- (regular) trips
- (regular) connections



$$\min c^{T} x$$

$$x(d, \delta^{+}(v)) = x(d, \delta^{-}(v)) \quad \forall v \in V, d \in D$$

$$x(\delta^{+}(t)) = 1 \qquad \forall t \in T$$

$$x \in \{0,1\}^{A \cup M}$$

#### The Coarse-to-Fine Method



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## Coarse-to-Fine Method: Layers

Problem specific layers:

- composition layer (with order and orientation, fine)
- configuration layer (with types and combinations, coarse)
- vehicle layer (individual vehicle flow, very coarse)

The layers are defined in terms of projections of hypergraphs that correspond to the projection of rows of the LP/IP.





## Coarse-to-Fine Method: General Setting

- Row and column index sets I = [m], J = [n]
- Matrix  $A \in \mathbb{R}^{I \times J}$
- Rhs  $b \in \mathbb{R}^{I}$
- Objective  $c \in \mathbb{R}^J$
- ► Linear Program

and its dual

$$\min c^T x$$
$$Ax = b$$
$$x \ge 0$$

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$$\max y^{T}b$$
$$y^{T}A = c^{T}$$
$$y \in \mathbf{IR}^{I}$$



- Aggregate/project the rows I of the (LP) by a problem specific coarsening projection []:  $I \rightarrow [I]$  (induces an equivalence relation)
- ► For a column vector  $v \in \mathbb{R}^{I}$  we define the coarsening of v as  $[v][i] \coloneqq (\min\{v_{k}: k \in I, [k] = [i]\}, \max\{v_{k}: k \in I, [k] = [i]\}) \cdot \tau(v, i)$ where  $\tau(v, i) \coloneqq |\{v_{k} \neq 0: [k] = [i]\}|$
- Coarse bimatrix [A], coarse dual vector  $[\pi]$
- Coarse objective function  $[c] \coloneqq c$  (no coarsening)



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$$\min[c]^T x, [A]x[=][b], x \in \mathbb{R}^{[J]},$$

where

$$[A]x[=][b]: \Leftrightarrow [b]_{[i]1} \le \sum_{j \in J} [A_{\cdot j}]_{[i]2} x_j, \ \sum_{j \in J} [A_{\cdot j}]_{[i]1} x_j \le [b]_{[i]2}, \ \forall [i].$$

Let  $P(A, b) \coloneqq \{Ax = b, x \ge 0\}$  and  $P([A], [b]) \coloneqq \{[A]x[=][b], x \ge 0\}.$ 

Lemma (B., Reuther, Schlechte, Weider [2015])

#### $P(A,b) \subseteq P([A],[b]).$



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## Coarse-to-Fine Method: Coarse Reduced Cost

• Multiplication of pairs  $(a_1, b_1), (a_2, b_2) \in \mathbb{R}^2$ :

 $(a_1, b_1), (a_2, b_2) \coloneqq \max\{a_1b_1, a_1b_2, a_2b_1, a_2b_2\}$ 

Coarse reduced cost for column j

$$\overline{[c_j]} \coloneqq [c_j] - [\pi]^T \cdot [a_j]$$

#### Lemma (B., Reuther, Schlechte, Weider [2015])

The coarse reduced cost always underestimates the original reduced cost

$$\overline{[c_j]} \coloneqq [c_j] - [\pi]^T \cdot [a_j] \le c_j - \pi^T \cdot a_j = \overline{c_j}.$$

Use the coarse reduced cost for pricing in the fine model.





#### Coarse-to-Fine Method: Example

(c1  $C_2$  $C_3$  $C_4$  $C_5$  $C_6$ C7 C8 C9 C10 C11 C12 C13 C14 C15 C16 C17 C18 C19 C20 C21 C22 C23 C24 C25) = 0 $+x_3$  $X_1$  $-x_5$  $-x_{7}$  $\pi_1$ ÷ = 0 $X_2$  $+x_4$  $-x_6 - x_7$  $\pi_2$ = 0 $X_5$  $+ x_7$  $\pi_3$  $-x_{11}$  $-x_{13}$ ; ÷ = 0 $x_6 + x_7$  $\pi_4$  $-x_{11}$  $-x_{13}$ = 0 $X_1$  $+ x_4$  $\pi_5$  $-x_{8}$  $-x_{10}$ =0  $x_2 + x_3$  $-x_9 - x_{10}$  $\pi_6$  $+x_{10}$ = 0X8  $-x_{12}-x_{13}$  $\pi_7$ = 0 $x_9 + x_{10}$  $-x_{12}-x_{13}$  $\pi_8$ =0  $+x_{13}-x_{14}$ x11  $\pi_{9}$ ÷ ÷  $+x_{13}-x_{14}$ =0 X11  $\pi_{10}$ = 0X14  $-x_{16}$  $\pi_{11}$ ÷ ÷ = 0X14  $-x_{17}$  $\pi_{12}$ = 0 $x_{12} + x_{13}$  $-x_{15}$  $\pi_{13}$  $x_{12} + x_{13}$ = 0 $-x_{15}$  $\pi_{14}$ = 0X15  $-x_{18}$  $\pi_{15}$ = 0X15 - X19  $\pi_{16}$ = 0 $+x_{19} - x_{20}$ X17  $\pi_{17}$  $+x_{20}$ =0  $-x_1 - x_2 - x_3 - x_4$  $\pi_{18}$ =0  $+x_{18}$ X16  $-x_{21}$  $\pi_{19}$ =0  $x_{21} - x_{22}$  $\pi_{20}$  $x_{22} - x_{23}$ = 0 $\pi_{21}$ =0  $x_{23} - x_{24}$  $\pi_{22}$  $x_{24} - x_{25} = 0$  $\pi_{23}$ 



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#### Coarse-to-Fine Method: Example







#### Coarse-to-Fine Method: Example


















# Coarse-to-Fine Method: Coarse Pricing



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- ► The solution of the root LP is 1.5-2x slower
- ► The solution of the node LPs is 10-20 x faster
- Memory consumption is much smaller





# Railway Constraints

Wagenstandanzeiger Gleis 11

Zeit	Zug		Richtung	G	F		D	C	B		IA
00.34	EN	Jan Konpura	Rosers Poenan GL Warstowe /			sa - 500 - 5 (500 -				ing .	
05.36	IC		BraunicTreng Magneturp Lengrg / Hulls Flugh	1	And and a second second						( <u></u> )+
06.21	ICE	Zugleburg in Harris	Criping Dible G Köln / Bonn Flughaten A bis C		Tage - Ser -	HELE DANS					) ->
06.40	IC		Köln Osnabrück Slad Bersheim Hengelo	C					and the sectors	Ref Institute	
07.45	IC	Dierstag bis Dorrienting	Amaterdam Centrael								
07.45	IC	Montag und Freitag	Bremen								
08.45	IC .		Bremen Verden Bremen Damerkost	C' I ILES						1270	
09.40	IC		Divertision Divertision Examination								
10.45	IC	Optimized	Dramon Oldantsung Einden			+					
11.40	1C		Biolefield Gotwelije Hanze			+				12	-
12.45	IC		Verden Verden Bremen Demenhon			+		- RIP - R			
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14.45	IC		Verden Diemen Demenhonst Offensione			+ @ 27 22	M . M	· PA 12" - 14			-
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16.45	IC		Bronsen Oktientiverg Einden Neuroklasisch Mole			+	- M - M			(IF (	
17.40	IC		Dorbreand Excert Dutations Köln			+ @				1	
18.45	IC		Verden Ekomen Demerkent			+ C II II				(E)= 6	



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# Maintenance: Service Intervals



Blue: timetabled trips Green: 4000 km treatment Dark gray: 8250 km treatment Yellow: 33000 km treatment Pink: 66000 km treatment Red: 198000 km treatment Light gray: 15 days treatment Turquoise: 30 days treatment



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# Railway Constraints

Wagenstandanzeiger Gleis 11

Zeit	Zug		Richtung	G	F	E	D	C	B		A
00.34	EN	Jan Kospuna	Respire Polenan GE Warstawa /	100		a - 20 - 20				ine and a second	
05.36	IC		Braunschweig Maginturp Leipzig / Halls Flugh	- 1 1							
06.21	ICE	Zugleburg in Harris	Leiping D bes G Köin / Bonn Flughaten A bis C		Tani ban Ban Ban Ban	Maria Barris Barris					)→ 
06.40	IC		OsnatinGos Bad Bentholm Hongelo Amsterdam Cantraal		-				Lan Initiation	ter bester	
07.45	IC	Develag bis Dovreising	Braman		-	9 8 8			150 E CO	121	
07.45	IC	Montag and Freitag	Braman			+					·
08.45	IC BH		Verden Bramen Demenhorst Obterburg			+ - 2 2				20	
09.40	IC		Deteriory Dortmanet Exercitionet			+ C A A			Nº 130		
10.45	IC	Dathiesland	Dramon Oldenburg Einden Rochtaich Mais			+ (* LE 11			1 A.C.		-
11.40	1C		Elisiefett Güterelut Hanza Destround			+ (* EB . S				1200	
12.45	IC		Verden Bremen Demoertoor Obterteen			• C B B			R* E.C*		
Re	<b>P</b> (	JUK	DHITN	/		+ @	- M- A			E (	<u></u>
14.45	IC		Verden Dremen Demenhonst Okteologra			+	· M · M	· PI · · · PI	20 M C*	137 6	
15.31	ICE	Zugtaliung in Harrin	O bis G Köln / Bonn Flughafen A bis C Köln	+						e Tas	
16.45	IC		Bromen Oklandung Einden Nordelisisch Male			+ C A A		- <b>2</b> - <b>2</b>	2* <b>1</b> 17*		
17.40	IC		Dorbreand Exear Dualstrag Kóln			+ @			** <b>M</b> (2*	13 C	
18.45	IC		Verden Ekomen Demerkont Osteren			+ @		. <b>R</b> 1*   <b>R</b>		E 2	



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# Parking: Keeping It Simple



Siding	Length/m	Feasible Assignments
1	570	XXX, YYY, XXX+YYY, YYY+YYY
2	480	XXX, YYY, YYY+YYY
3	430	XXX, YYY, YYY+YYY
4,5,6	420	XXX, YYY, YYY+YYY
7	410	XXX, YYY, YYY+YYY
8	390	XXX, YYY
9,10,11	240	YYY
12,13,14	210	YYY



### Vehicle Rotation Planning Model

$$\min \sum_{\alpha \in H} c_{\alpha} x_{\alpha}, \qquad (objective)$$

$$\sum_{\alpha \in H(t)} x_{\alpha} = 1 \qquad \forall t \in T, \qquad (1)$$

$$\sum_{\alpha \in H(|v|)^{in}} x_{\alpha} = \sum_{\alpha \in H(v)^{out}} x_{\alpha} \qquad \forall v \in V, \qquad (2)$$

$$w_{a}^{I} \leq \sum_{\alpha \in H(a)} U_{I} \times_{\alpha} \qquad \forall a \in A, I \in L,$$
 (3)

$$\sum_{\boldsymbol{a}\in\boldsymbol{A}(\boldsymbol{v})^{\text{out}}} w_{\boldsymbol{a}}^{\boldsymbol{I}} - \sum_{\boldsymbol{a}\in\boldsymbol{A}(\boldsymbol{v})^{\text{in}}} w_{\boldsymbol{a}}^{\boldsymbol{I}} = \sum_{\boldsymbol{\alpha}\in\boldsymbol{H}(\boldsymbol{v})^{\text{out}}} r_{\boldsymbol{I}}^{\boldsymbol{v}}(\boldsymbol{\alpha}) \times_{\boldsymbol{\alpha}} \quad \forall \boldsymbol{v}\in\boldsymbol{V}, \ \boldsymbol{I}\in\boldsymbol{L},$$
(4)

$$\sum_{a \in \boldsymbol{H}} \boldsymbol{r}_{\boldsymbol{b}}(a) \boldsymbol{x}_{a} \leq \boldsymbol{U}_{\boldsymbol{b}} \qquad \forall \boldsymbol{b} \in \boldsymbol{B},$$
(5)

$$x_{\mathfrak{a}} \in \{0, 1\}$$
  $\forall \mathfrak{a} \in H,$  (6)

$$w'_{a} \in [0, U_{I}] \subset \mathbb{Q}_{+} \qquad \forall a \in A, I \in L$$
(7)

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# Real World Example: Scenario 1

Input	#	Objective	Goal
Timetabled trips	798	Coverage	100%
Connections	171	Rows	Minimum
<ul> <li>Maintenance interval</li> <li>Small: every 12500 km @ 1 depot</li> <li>Monthly: every 25000 km @ 1 depot</li> <li>Big: every 50000 km @ 1 depot</li> </ul>	3	No of. maintenance services	Minimum
Stations	14		
Depots	7		

Objective	Reference solution	VS-OPT rail
Rows	20 + 300 km deadhead	19 + 300 km deadhead
CPU time (hh:mm)	— <b>:</b> —	00:20



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# Real World Example: Scenario 2

Input	#	Objective	Goal	
Timetabled trips	1292	Trip coverage	100%	
Connections	1009	Rows	Minimum	
<ul> <li>Maintenance intervals</li> <li>Refuel: every 600 km @ 10 depots</li> <li>Small: every 15000 km @ 1 depot</li> <li>Big: every 60000 km @ 1 depot)</li> </ul>	3	No of maintenance services	Minimum	
Stations	26			
Depots	34			

Objective	Reference solution	VS-OPT rail
Rows	29 + 5500 km deadhead	26 + 3300 km deadhead
CPU time (hh:mm)	—:—	08:48

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# Delay Resistant Train Rotations







#### Measuring Robustness as Expected Propagated Delay

Initially set arrival delay  $AD_t = 0$ .



For a trip *t*:

- C<sub>t</sub> set of connecting trips
- b<sub>s,t</sub> stop over time
- D<sub>t</sub> primary delay (e.g. breakdowns, disruptions)
- c turn over time ( e.g. cleaning, crew changes)





# Minimizing the Expected Propagated Delay

- Computing the propagated delay distribution requires a convolution
- Approximate primary delay using discrete random variables
- Numerical effort is quadratic in the number of discretization intervals
- Approach: penalize small buffers, verify EPD



Approximation of  $X \sim \operatorname{Exp}(1)$ 

#### EPD: -9% EPD for +5% Cost?



# The Price of Regularity: Case Study

#### Real world scenario

- 670 timetabled trips (127 trains)
- ► 52 locations
- vehicle compositions of size at most two
- 4 946 356 hyperarcs

Bi-criteria objective function

- Minimize operational cost including
  - cost for rolling stock
  - cost for deadhead trips
  - cost for additional turn around trips
  - cost for violating planned turn times
- Maximize regularity (i.e., minimize irregularity penalty R)
- Weighted sum method



## PoR: Complex Coupling Requirements



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# PoR: Complex Coupling Requirements (cont'd)



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# PoR: Additional Turn-around Trips





# PoR: Additional Turn-around Trips





# PoR: Additional Turn-around Trips





#### **PoR:** Results



# Thank you for your attention



#### THE POWER OF COOPERATION

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